Università
della
Svizzera
italiana

**Faculty
of
Informatics**

CMIS 2026

Contact Mechanics International Symposium

21-24 April 2026
Continental Park Hotel Lugano

History

1992 — **Lausanne, Switzerland**
Chairman:
Alain CURNIER

1994 — **Carry le Rouet, France**
Chairmen:
Michel RAOUS, Jean-Jacques MOREAU & Michel JEAN

2001 — **Peniche, Portugal**
Chairmen:
Joao A.C. MARTINS & Manuel D.P. MONTEIRO MARQUES

2005 — **Hannover, Germany**
Chairman:
Peter WRIGGERS

2009 — **Chania, Greece**
Chairman:
Georgios E. STRAVROULAKIS

2012 — **Cargese, France**
Chairmen:
Michel RAOUS & Peter WRIGGERS

2014 — **Abu Dhabi, United Arab Emirates**
Chairman:
Tod LAURSEN

2016 — **Warsaw, Poland**
Chairman:
Stanislaw STUPKIEWICZ

2018 — **Oropa (Biella), Italy**
Chairmen:
Giorgio ZAVARISE, Michel RAOUS & Peter WRIGGERS

2022 — **Chexbres, Switzerland**
Chairman:
Jean-François MOLINARI

2024 — **Lyon, France**
Chairmen:
David DUREISSEIX, Anthony GRAVOUIL, Michel RAOUS & Yves RENARD

2026 — **Lugano, Switzerland**
Chairpersons:
Rolf KRAUSE, Patrick ZULIAN, Marco FAVINO & Maria Giuseppina Chiara NESTOLA

CMIS 2026

The 12th Contact Mechanics International Symposium (CMIS) aims at gathering researchers interested in a broad range of topics in theoretical, mathematical, computational and experimental aspects of contact mechanics.

The symposium offers a broad and inclusive program designed to foster interaction and collaboration across the diverse communities engaged in contact mechanics.

Topics include, but are not limited to:
- Models, friction laws, wear, tribological modeling, cohesive zone models, rolling;
- Emerging computational approaches: methods, algorithms, and numerical analysis;
- Mathematical analysis of contact problems;
- Dynamic contact problems and instabilities;
- Nano- and micromechanics of contact; multiscale methods;
- Multiphysics and thermomechanical coupling;
- Contact in granular materials and rigid body systems;
- Applications in mechanical and civil engineering, biomechanics, and geomechanics;
- Experimental investigations in contact mechanics.

New in this edition:
- Contact modeling in computer graphics and animation;
- High-performance computing (HPC) strategies for large-scale contact problems.

True to the CMIS tradition, the venue has been carefully selected to provide a full board accommodation in a stimulating environment for scientific exchange and in-depth discussions. The Symposium will be held at the Continental Parkhotel which is a historic hotel in Lugano, built in stages between 1870 and 1906 by the Hirth-Wyss family and later by Sir Johann Helmsauer.

Organizing institutions

We are grateful to USI for their support and help in organizing the conference, and to KAUST for their support.

Università della Svizzera italiana

جامعة الملك عبدالله للعلوم والتقنية
King Abdullah University of Science and Technology

Sponsors

We extend our deepest gratitude to our generous sponsors for their invaluable support and commitment to advancing scientific research and collaboration. Your contributions have been essential in bringing together experts, researchers, and innovators from around the world to share ideas, inspire progress, and shape the future of science.

Swiss National Science Foundation

prodir®

Committee

It is our great pleasure to welcome you to the Contact Mechanics International Symposium (CMIS 2026), hosted at Università della Svizzera italiana (USI) in Lugano, with the participation of King Abdullah University of Science and Technology (KAUST).

On behalf of USI, KAUST, and personally, I am delighted to greet all participants and to host this edition of CMIS in Lugano. It is both an honor and a privilege to bring together the international contact mechanics community in this unique setting.

The scientific program reflects the breadth and vitality of the field. It spans a wide range of topics—from tribology and experimental studies to theoretical and computational contact mechanics—creating a scientifically rich platform for exchange. We are particularly pleased that CMIS 2026 brings together researchers from diverse backgrounds, fostering interactions that we hope will spark new ideas, collaborations, and perspectives.

We are grateful for the opportunity to host this symposium in Ticino and to welcome you to Lugano. We warmly invite you not only to engage with the scientific program, but also to enjoy the hospitality of Ticino, its stunning landscapes, and the unique blend of Swissness and Italianity that characterizes this region.

We wish you a stimulating and inspiring conference.

Conference organizers

Prof. Dr. Rolf Krause
Euler Institute, Faculty of Informatics, Università della Svizzera italiana, Switzerland;
Computer, Electrical and Mathematical Sciences and Engineering Division,
King Abdullah University of Science and Technology, Saudi Arabia

Dr. Patrick Zulian
Euler Institute, Faculty of Informatics, Università della Svizzera italiana, Switzerland;
UniDistance Suisse, Brig, Switzerland

Dr. Marco Favino
Euler Institute, Faculty of Informatics, Università della Svizzera italiana, Switzerland;
UniDistance Suisse, Brig, Switzerland

Dr. Maria Giuseppina Chiara Nestola
Euler Institute, Faculty of Informatics, Università della Svizzera italiana, Switzerland

Scientific committee

Acary Vincent, INRIA, Grenoble, France
Ballard Patrick, CNRS, Paris, France
Carbone Giuseppe, Politecnico di Bari, Italy
Dostal Zdenek, Technical University of Ostrava, Czech Republic
Favino Marco, Università della Svizzera italiana, Lugano, Switzerland
Gay Neto Alfredo, Universidade de São Paulo, Brasil
Grillo Alfio, Politecnico di Torino, Torino, Italy
Haslinger Jaroslav, Charles University, Prague, Czech Republic
Krause Rolf, King Abdullah University of Science and Technology, Thuwal, Kingdom of Saudi Arabia
Laursen Tod, American University of Sharjah, Sharjah, UAE
Lebon Frederic, CNRS-AMU-ECM, Marseille, France
Molinari Jean-François, EPFL, Lausanne, Switzerland
Nackenhorst Udo Leibniz Universität Hannover, Germany
Nestola Maria, Università della Svizzera italiana, Lugano, Switzerland
Paggi Marco, IMT School for Advanced Studies, Lucca, Italy
Panozzo Daniele, Courant Institute of Mathematical Sciences, New York University, USA
Paoli Laetitia, University of Saint Etienne, France
Popov Valentin, Technische Universität Berlin, Germany
Popp Alexander, University of the Bundeswehr Munich, Germany
Puso Mike, Livermore National Laboratory, CA, USA
Raous Michel, CNRS-AMU-ECM, Marseille, France
Renard Yves, INSA Lyon, France
Sacco Elio, University of Naples, Italy
Scheibert Julien, CNRS-ECL, Lyon, France
Schneider Teseo, University of Victoria, Victoria BC, Canada
Stavroulakis Georgios, Technical University of Crete, Chania, Greece
Stolz Claude, EC Nantes, France
Stupkiewicz Stanislaw, Polish Accademy of Sciences, Warsaw, Poland
Wriggers Peter, Leibniz Universität Hannover, Germany
Zavarise Giorgio, Politecnico di Torino, Torino, Italy
Zulian Patrick, Università della Svizzera italiana, Lugano, Switzerland

Foreword

As I mentioned during my participation in the opening session, CMIS is much more than a classical symposium: it is a true international Thinktank dedicated to contact mechanics, offering not only presentations of recent results, but also a forum for discussion and exchange within our community.

This is made possible by its specific organization: a single location for conferences and accommodation, no parallel sessions, a wide range of topics related to contact mechanics, and registration fees kept as low as possible.

This booklet presents the abstracts of the presentations outlining the recent results from our community.
The quality of the forum and the scientific exchanges is reflected in the connections forged between participants, making

CMIS an essential event for the contact mechanics community.

Michel Raous
DRE CNRS
LMA - Marseille - France

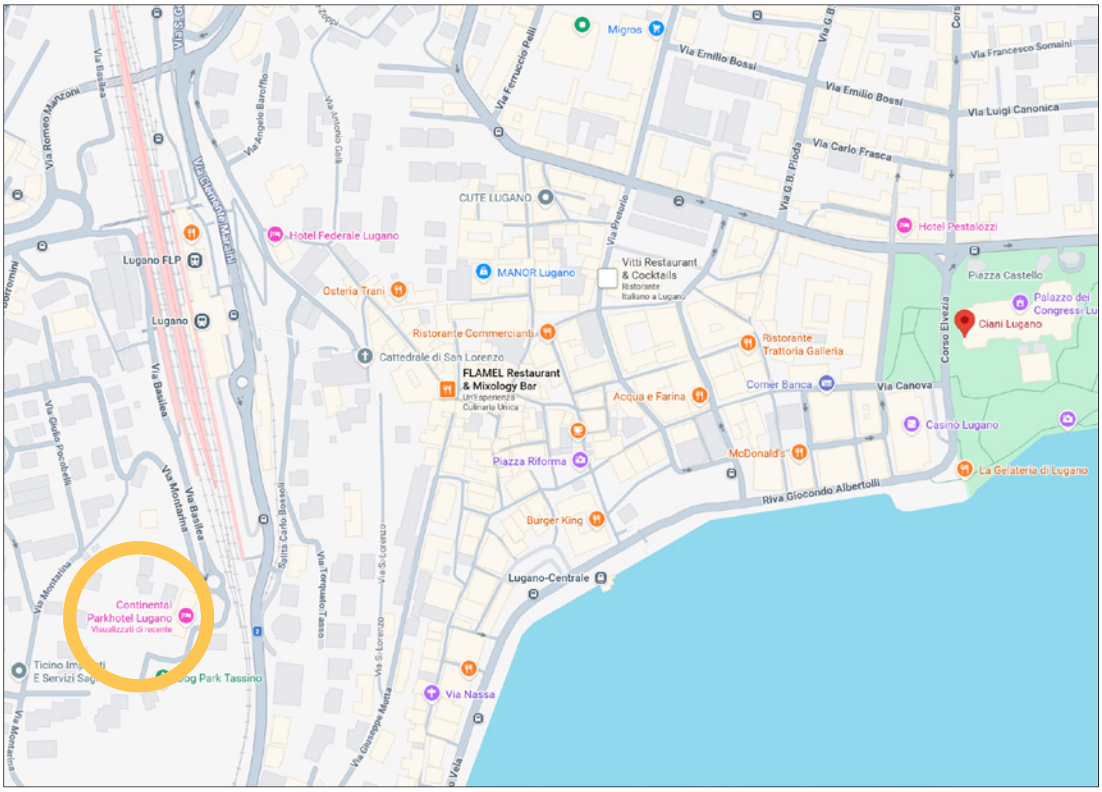
Practical Informations

Contact

If you need assistance, please speak to Patrick Zulian and Marco Favino (local CMIS organizers) who will be happy to assist you.

Venue / Directions

Continental Parkhotel Lugano
Via Basilea 28, 6900 Lugano



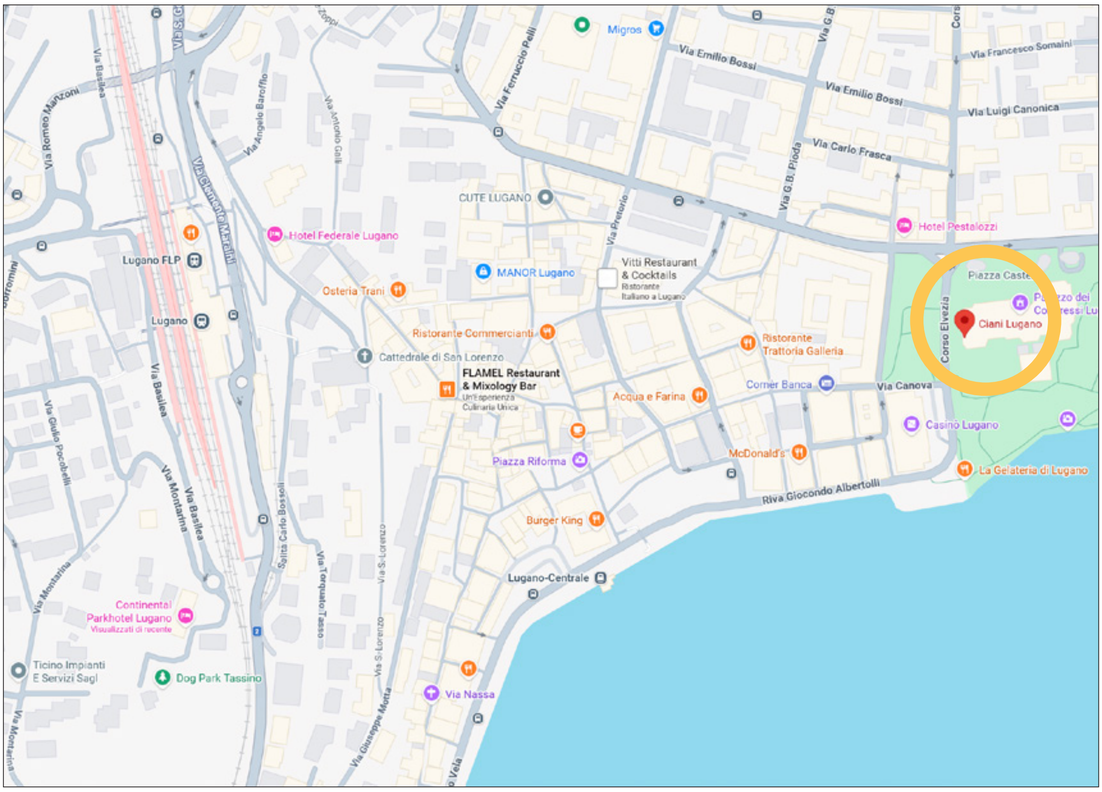
Smoking Policy

The Continental Parkhotel is a designated no-smoking building. If you wish to smoke, please do so outside the buildings.

Social Banquet

Ristorante Ciani Lugano
Piazza Indipendenza 4, 6900 Lugano

Located on the ground floor of the Palazzo dei Congressi, Ristorante Ciani Lugano enjoys a prime location just steps from the city center and close to a large and convenient parking garage.
It's all set in a unique and stunning setting, enhanced by the spacious terrace nestled in the greenery of the park overlooking the lake.



Ciani Lugano

Program

Tuesday 21 April 2026

16.00 - 21.00	**Registration and welcome dinner**

Wednesday 22 April 2026

07.15 - 08.10	**Breakfast**
08.30 - 08.45	Welcoming speech
08.45 - 09.00	Opening speech
09.00 - 09.50	**Keynote 1 — Patrick Ballard** Transition from Continuous to Jumping Solutions in Quasi-static Elastic Contact Problems with Coulomb Friction: the Prediction of the Onset of Brake Squeal
09.50 - 10.50	**Session 1** **1. Acary Vincent** Interior-Point and Asymptotic Numerical Methods for Frictional Contact Problems **2. Alfredo Gay Neto** Multiple Pointwise Contact Search in Engineering Applications Considering Enforcement by a Hybrid-Barrier Interface **3. Ali Maghami** Toward Multi-Scale Learning of Adhesive Contact Forces: From Single Asperities to Rough Surfaces
10.50 - 11.10	**Coffee break**
11.10 - 12.50	**Session 2** **1. Andrzej Myslinski** Adhesion Modelling in Wheel-Rail Frictional Contact **2. Antonio Papangelo** Recent Advancement in Understanding Vibroadhesion: Experiments and Modelling **3. Baptiste Boulet** Physical Origin of Friction Relaxation After Interrupted Sliding **4. Gabriele Marchi** Shifted Penalty Multigrid Method for Contact **5. Cédric Pozzolini** Application of the Nitsche Method to Three Industrial 3D Elastoplastic Frictional Contact Problems
13.05 - 14.25	**Lunch**

14.35 - 15.25	**Keynote 2 — Daniele Panozzo** Geometric Potential and Geometric Predicates for Unconditionally Robust Elastodynamics Simulation
15.25 - 16.25	**Session 3** **1. Daniel Wolff** Energy-Based Physics-Informed Neural Networks for Nonlinear Contact Mechanics **2. Davy Dalmas** Advancing Parameter Extraction in Adhesive Contact Mechanics: A Method for Estimating the Tabor Parameter Across Adhesive Re- **3. Gabor Csernak** Monostable, Bistable, and Tristable Scenarios in a Simple Mechanical Model With Lugre Friction
16.25 - 16.45	Coffee break
16.45 - 18.05	**Session 4** **1. Carmine Putignano** A Variational Energy-Based Boundary Element Approach for 3D Adhesive Contacts **2. Ivan Argatov** Liquid Hertz Impact **3. Ivo Steinbrecher** Mixed-Dimensional Beam-To-Solid Interaction: From Embedded Fibers to Contact **4. Julien Scheibert** An Optimization-Based Framework to Design Metainterfaces With Tailored Friction Faws
18.25 - 21.45	**Poster session** **Apero** **Dinner**

Thursday 23 April 2026

07.15 - 08.10	**Breakfast**
08.30 - 09.20	**Keynote 3 — Peter Wriggers** Recent Advances in Third Medium Contact
09.20 - 10.40	**Session 5** 1. **Katarina Tutić** Analytical Modeling and Experimental Validation of Ball–beam Impact Dynamics Using a Nonsmooth Contact Dynamics Framework 2. **Lucas Frérot** Evolution of the Contact Between Rough Viscoelastic Solids After Decreasing Loads: Memory Erasure and Monotonic Increase 3. **Mille Antoine** Effect of Velocity Ratio on Slip Field During Oblique Landing of Elastomer Spheres 4. **Nicolas Durand** Semi-Analytical Model of Periodic Coated Viscoelastic Contact
10.40 - 11.10	**Coffee break**
11.10 - 12.50	**Session 6** 1. **Pietro Davide Maddio** A Frictional Contact Formulation for Planar Mechanisms 2. **Qiang Li** Partial Slip in Contact of an Elastic Quarter-Space 3. **Qingfeng Lou** Apoint-Wise Complementarity Model for Frictional Contact Problems 4. **Rolf Stenberg** Finite Element Methods for Elastic Contact: Augmented Lagrangian and Nitsche 5. **Sauer Roger** A monolithically Coupled Chemo-Mechano-Thermodynamical Contact Formulation
13.05 - 14.25	**Lunch**
14.35 - 15.25	**Keynote 4 — Mike Puso** Where is contact now?
15.25 - 16.25	**Session 7** 1. **Schorsch Matthieu** Extending Nitsche's Method for the Unilateral Contact of Beams 2. **Tiago Silva Sabino** A Mortar-Type Computational Approach for the Simulation of Wear in the Presence of Third Bodies and Transfer Film 3. **Valentine Rey** Comparison of Three Algorithms on Frictional Contact Problem With Two Distinct Solutions
16.25 - 16.45	**Coffee break**
16.45 - 17.45	**Session 8** 1. **Vladislav Mantic** A Generalised Comninou Contact Model for Interface Cracks in Anisotropic Bimaterials 2. **Yves Renard** Dynamics With Impact of Elastic Solids : Issues, Classical Schemes and Recent Advances 3. **Giacomo Petraglia** A fractional Model of Finite Viscoelasto-Plasticity Including Damage Through the Phase Field Technique
18.30 - 23.00	**Walk to Parco Ciani and Social dinner: Ristorante Ciani Lugano**

Friday 24 April 2026

07.15 - 08.10	**Breakfast**
08.30 - 09.20	**Keynote 5 — Anka Chen** Toward Real-Time, Stable, and Physically Accurate Simulation: Vertex Block Descent and Beyond
09.20 - 10.40	**Session 9 (4 talks)** **1. Guillaume Mestdagh** Dynamic Cable Simulation for Ropeway Transport Systems **2. Jonas Breuling** Efficient Mixed Rod Finite Elements for Rockfall Protection Ring-Net Barriers With Frictional Contact **3. Moritz Billen** Skin-Pass Rolling as a Fluid-Structure-Contact Interaction Problem **4. Rawane Mansour** Variational Formulation and Numerical Resolution of Persistent Adhesive Contact in Hyperelastic Materials
10.40 - 11.10	**Coffee break**
11.10 - 12.50	**Session 10** **1. Emile Hohnadel** Accurate Contact Detection for the Predictive Simulation of Fibre Assemblies **2. Alessandro Cammarata** A multiarea Frictionless Contact Method for Planar Mechanism **3. Ayman Khaddari** Influence of Numerical Schemes on the Apparent Dynamics of the Contact Between a Viscoelastic Beam and Two Rigid Stops **4. Martin Weiser** Multilevel Augmented-Lagrangian Methods for Overconstrained Contact Discretizations
13.05 - 14.25	**Lunch**
14.35 - 15.25	**Keynote 6 — Jean-François Molinari** Space under stress
15.25 - 17.05	**Session 11** **1. Jozsef Kovecses** Reduced-Order Compliant Wheel Obstacle Contact Interaction for Rover Mobility With Experimental Validation **2. Paulo Ricardo Ferreira Rocha** Surface Pattern Design Using the Boundary Element Method and Bayesian Optimisation **3. Alessandro Gianmarini** A Mathematical Framework for Addressing Boundary Conditions in Remodeling, Hydrated Soft Tissues **4. Rolf Krause** Solving Frictional Contact Problems in Space and Time at Scale **5. Patrick Zulian** Immersed Methods for Fluid-Structure-Contact Interaction
17.05 - 17.15	**Closing remarks**

Invited Speakers

Patrick Ballard got an engineering degree from École Polytechnique (near Paris, France) in 1987 and a PhD in mechanical engineering from École Polytechnique in 1991. He held several CNRS permanent research positions in École Polytechnique (1991->2004), in Marseille (2004->2015). Since 2015, he has been a CNRS senior researcher at Institut Jean le Rond d'Alembert, Sorbonne Université, Paris, France.

Daniele Panozzo is a Full Professor of Computer Science and Vice-Dean of Academic Affairs at the Courant Institute School of Mathematics, Computing, and Data Science. Prior to joining NYU he was a postdoctoral researcher at ETH Zurich (2012–2015).
Daniele earned his PhD in Computer Science from the University of Genova (2012) and his doctoral thesis received the EUROGRAPHICS Award for Best PhD Thesis (2013). He received the EUROGRAPHICS Young Researcher Award in 2015, the NSF CAREER Award in 2017, and a Sloan Research Fellowship in 2020. Daniele's research group is leading the development of PolyFEM (https://polyfem.github.io), a simple and robust finite element library, and wild meshing (https://github.com/wildmeshing), a 2D and 3D robust meshing library. Daniele initiated the Graphics Replicability Stamp (http://www.replicabilitystamp.org), which is an initiative to promote reproducibility of research results and to allow scientists and practitioners to immediately benefit from state-of-the-art research results. His research interests are in finite element simulation, digital fabrication, and geometry processing.

Professor Dr.-Ing. habil. **Peter Wriggers** studied Civil Engineering at the University Hannover, he obtained his Dr.-Ing degree at the University Hannover in 1980 on "Contact-impact problems". From 1983-84 he was Visiting Scholar at the UC Berkeley, USA. In 1990 he was appointed as Full Professor at the Institute of Mechanics at TH Darmstadt. In 1998 Prof. Wriggers changed to the Leibniz University Hannover where he held the chairs for Mechanics in Civil Engineering and in Mechanical Engineering. Since April 2022 he is Emeritus Professor at Leibniz Universität Hannover and was awarded the status of "Leibniz Emeritus". From 2003 to 2004 he held the position of "Linkage Professor" at the University of Newcastle in NSW, Australia. From 2014 to 2021 he was Vice-President for Research of the Leibniz University Hannover.
Peter Wriggers is member of the "Braunschweigische Wissenschaftliche Gesellschaft", the Academy of Science and Literature in Mainz, the German National Academy of Engineering "acatech" and corresponding member of the National Academy of Croatia. He was President of GAMM, President of GACM and Vice-President of IACM. Furthermore, he acts as Editor-in-Chief for the International Journal "Computational Mechanics" and was from 2014 to 2025 Editor-in-Chief of "Computational Particle Mechanics". He was awarded the Fellowship of IACM and received the "Computational Mechanics Award", the "IACM Award" and the "Gauss-Newton Medal" of IACM, the "Euler Medal" of ECCOMAS, the Grand Prize of the Japan Society (JSCES), the "Zienkiewicz Medal" of the Polish Association (PACM) as well as three honorary degrees from the Universities of Poznan, ENS Paris-Saclay and TU Darmstadt.

Mike Puso graduated from UC Davis in 1994 and has been a code developer and researcher in computational mechanics for 31 years. He specializes in contact methods, element technologies, time integration schemes and material modeling. He enjoys golf, skiing, baseball, music, history, hanging out with family and playing with his dog.

Anka Chen is a Senior Research Scientist at NVIDIA, where she is a core contributor to NVIDIA's Newton physics engine. She is a co-creator of the Vertex Block Descent (VBD) solver and the Offset Geometric Contact (OGC) method. Her work focuses on parallel solutions for implicit time integration, with an emphasis on robust and efficient handling of elasticity, collisions, and rigid body dynamics. She received her PhD in Computer Graphics from the University of Utah, where her dissertation received the SCA Best Doctoral Dissertation Award.

Professor **Jean-François Molinari** is the director of the Computational Solid Mechanics Laboratory (http://lsms.epfl.ch) at EPFL, Switzerland. He holds an appointment in the Civil Engineering institute, which he directed from 2013 to 2017, and a joint appointment in the Materials Science institute. He started his tenure at EPFL in 2007, and was promoted to Full Professor in 2012. He is currently an elected member of the Research Council of the Swiss National Science Foundation in Division 2 (Mathematics, Natural and Engineering Sciences), and co editor in chief of the journal Mechanics of Materials.
J.F. Molinari graduated from Caltech, USA, in 2001, with a M.S. and Ph.D. in Aeronautics. He held professorships in several countries besides Switzerland, including the United States with a position in Mechanical Engineering at the Johns Hopkins University (2000-2006), and France at Ecole Normale Supérieure Cachan in Mechanics (2005-2007), as well as a Teaching Associate position at the Ecole Polytechnique de Paris (2006-2009).
The work conducted by Prof. Molinari and his collaborators takes place at the frontier between traditional disciplines and covers several length scales from atomistic to macroscopic scales. Over the years, Professor Molinari and his group have been developing novel multiscale approaches for a seamless coupling across scales. The activities of the laboratory span the domains of damage mechanics of materials and structures, nano- and microstructural mechanical properties, and tribology.

Wednesday 22 April 2026

Transition from Continuous to Jumping Solutions in Quasi-static Elastic Contact Problems with Coulomb Friction: the Prediction of the Onset of Brake Squeal

Patrick Ballard (joint work with Flaviana Iurlano)

I will revisit the classical quasi-static elastic contact problem with Coulomb friction from a mathematical perspective (that is, existence and qualitative properties of solutions).

Let $\Omega \subset \mathbb{R}^N$ be a smooth bounded open set and $\partial\Omega = \Gamma_U \cup \Gamma_T \cup \Gamma_C$ a partition of its boundary into three disjoint parts (Dirichlet, Neumann and contact). The outward unit normal is denoted by $\mathbf{n}$. The framework is that of linear elasticity, so that the stress $\boldsymbol{\sigma}(\mathbf{u})$ is a linear function of the displacement $\mathbf{u}$. The boundary traction $\mathbf{t} := \boldsymbol{\sigma}(\mathbf{u})\,\mathbf{n}$ can be split into normal and tangential parts $\mathbf{t} = t_{\mathrm{n}}\mathbf{n} + \mathbf{t}_{\mathrm{t}}$ with respect to the reference configuration Ω. Considering the time interval $s \in [0, S]$, we are given time-varying body forces $\mathbf{F}(s) : \Omega \to \mathbb{R}^N$ and surface forces $\mathbf{T}(s) : \Gamma_T \to \mathbb{R}^N$. The quasi-static elastic contact problem with Coulomb friction is formally that of finding a time-varying displacement $\mathbf{u}(s) : \Omega \to \mathbb{R}^N$ satisfying a given initial condition and, for all $s \in [0, S]$:

$$\begin{cases} \operatorname{div}\boldsymbol{\sigma}(\mathbf{u}) + \mathbf{F} = \mathbf{0}, & \text{in } \Omega, \\ \mathbf{u} = \mathbf{0}, & \text{on } \Gamma_U, \\ \boldsymbol{\sigma}(\mathbf{u})\,\mathbf{n} = \mathbf{T}, & \text{on } \Gamma_T, \\ u_{\mathrm{n}} - g \leq 0, \qquad t_{\mathrm{n}} \leq 0, \qquad t_{\mathrm{n}}\,(u_{\mathrm{n}} - g) = 0, & \text{on } \Gamma_C, \\ \forall \mathbf{v} \in \mathbb{R}^N, \qquad \mathbf{t}_{\mathrm{t}} \cdot (\mathbf{v} - \dot{\mathbf{u}}_{\mathrm{t}}) - f t_{\mathrm{n}}(|\mathbf{v}| - |\dot{\mathbf{u}}_{\mathrm{t}}|) \geq 0, & \text{on } \Gamma_C. \end{cases}$$

Here, $\dot{\mathbf{u}}$ stands as usual for the time derivative of $\mathbf{u}$. The first line of the conditions on Γ_C are the Signorini conditions for unilateral contact with a rigid obstacle whose shape is encoded by the given gap function $g : \Gamma_C \to \mathbb{R}$. The second line of the conditions on Γ_C is a synthetic formulation of the ubiquitous Coulomb law of dry friction, with given friction coefficient $f \geq 0$.

The quasi-static elastic contact problem with Coulomb friction is invariant under monotone reparameterization of time. It therefore falls into the class of *rate-independent processes*, such as perfect elastoplasticity [3] or brittle fracture [4]. A natural mathematical framework for the analysis of rate-independent processes is that of functions with *bounded variation in time*, both for the solutions and the loads. Such functions may have jump discontinuities in time. Therefore, a natural question to ask about rate-independent processes is whether solutions can have spontaneous jumps in time, even when the loads are smooth: do absolutely continuous loads always yield absolutely continuous solutions, or not?

- In the case of brittle fracture, finite jumps (in time) in the solution produced by infinitesimal changes in the load have been evidenced in the quasi-static theory. This was interpreted as the fact that unstable crack propagation makes brittle fracture inherently a dynamic process, which should be analyzed in the framework of elastodynamics. Besides, this is evidenced by the sound emitted by crack propagation in glass.

- In the case of perfect elastoplasticity, absolutely continuous loads always yield absolutely continuous solutions [3]. Besides, it is common experience that the plastic bending of a metallic spoon is silent.

After having provided the appropriate formulation of the quasi-static elastic contact problem with Coulomb friction in the framework of functions with bounded variation in time,

- I will present examples (large friction coefficients) of solutions with spontaneous jumps in time, while no continuous-in-time solution exists, despite absolutely continuous loads,
- I will present a theorem (only in 2D, for now) ensuring that in the case of small friction coefficients, absolutely continuous loads always yield absolutely continuous solutions,
- I will present the optimal condition on the friction coefficient separating these two regimes.

These results extend the early analysis of Klarbring [5] of the two-degrees-of-freedom problem where the elasticity operator is replaced by the 2×2 positive definite symmetric stiffness matrix:

$$\mathbf{K} = \begin{pmatrix} k_{\mathrm{nn}} & k_{\mathrm{nt}} \\ k_{\mathrm{nt}} & k_{\mathrm{tt}} \end{pmatrix}.$$

Klarbring studied existence and uniqueness of solution for the *rate problem* (given a current configuration at time s and a velocity of the loads, find the velocity of the solution at time s). He showed that:

- if $f|k_{\mathrm{nt}}| < k_{\mathrm{tt}}$ (Klarbring's condition), then the rate problem always has a unique solution,
- if $f|k_{\mathrm{nt}}| \geq k_{\mathrm{tt}}$, then the rate problem can have either no solution or several solutions.

Formulating Klarbring's problem in the bounded variation framework, I will show that the two-degrees-of-freedom quasi-static problem with arbitrarily large friction coefficient always has solutions with bounded variation, possibly with spontaneous jumps in time (in the cases where Klarbring's rate problem has no solution). I will also prove that, under Klarbring's condition $f|k_{\mathrm{nt}}| < k_{\mathrm{tt}}$, absolutely continuous loads always yield absolutely continuous solutions (no spontaneous jumps in time, if the loads have no jumps). Hence, the new results I will present can be seen as a generalization of Klarbring's condition to the case of a continuum, paving the road to practical design against the onset of friction-induced vibrations, such as brake squeal.

References

[1] P. Ballard and F. Iurlano (2024), Existence results for the time-incremental elastic contact problem with Coulomb friction in 2D. *Mathematical Models and Methods in Applied Sciences*, **34**, No 12, pp 2217–2263.

[2] P. Ballard and F. Iurlano (2025), Transition from Continuous to Jumping Solutions in 2D Quasi-static Elastic Contact Problems with Coulomb Friction: the Mathematics Underlying the Onset of Brake Squeal (currently under review).

[3] G. Dal Maso, A. DeSimone and M.G. Mora (2006), Quasistatic evolution problems for linearly elastic–perfectly plastic materials. *Archive for Rational Mechanics and Analysis* **180**, pp 237–291.

[4] G.A. Francfort and J.J. Marigo (1998), Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids* **46**, No 8, pp 1319–1342.

[5] A. Klarbring (1990), Examples of non-uniqueness and non-existence of solutions to quasistatic contact problems with friction. *Ingenieur-Archiv* **60**, No 8, pp 529–541.

Interior-Point and Asymptotic Numerical Methods for Frictional Contact Problems

Vincent Acary, and Paul Armand

INRIA. TRIPOP team. Centre de l'Université Grenoble Alpes, France

Unilateral contact problems with Coulomb friction remain a central challenge in computational contact mechanics due to their intrinsic nonsmoothness, nonconvexity, and frequent rank deficiency arising from redundant constraints. Such difficulties are particularly acute in rigid multibody systems, granular media, and large assemblies of contacting bodies, FEM with structural elements (beams, plates and shells), where classical second-order methods often fail to converge robustly, while first-order methods—although reliable—suffer from slow convergence and limited accuracy.

In this work, we propose a *scalable second-order solution strategy* for frictional contact mechanics that combines a *primal–dual interior point method (IPM)*[1] with a path-following strategy based on the *Asymptotic Numerical Method (ANM)*[4]. The resulting approach, referred to as **IPM-ANM**, is specifically designed to address the challenges of nonsmoothness, hyperstaticity, and non-monotone solution paths that arise in discrete Coulomb friction models.

SOCCP formulation. Following standard time and space discretization of mechanical systems with unilateral contact and Coulomb friction, the governing equations are formulated as a Second-Order Cone Complementarity Problem (SOCCP)[3]. The contact reactions and relative velocities at each contact point are constrained by second-order (Lorentz) cones representing the Coulomb friction law. The resulting system is nonlinear, nonsmooth, and generally rank-deficient, as the contact configuration matrix is often not of full row rank. This framework allows the problem to be cast into a form amenable to interior-point techniques, while preserving the mechanical interpretation of sticking, sliding, and take-off contact states.

Interior-point framework and central path. The proposed solution strategy is based on a primal–dual interior-point method inspired by Mehrotra's predictor–corrector algorithm. A key theoretical contribution of this work is the proof of the existence of an analytic central path associated with the perturbed complementarity system. Under a Slater-type condition, we show that for any positive barrier parameter, the perturbed system admits a solution and that accumulation points of this path correspond to solutions of the original frictional contact problem.

This analysis provides not only a solid mathematical foundation for the interior-point algorithm, but also an alternative proof of solution existence for the SOCCP formulation of Coulomb friction. The interior-point iterations rely on semi-smooth Newton steps to handle the nonsmooth terms associated with friction, while maintaining iterates strictly within the interior of the friction cones.

Limitations of classical path-following. Despite its strong theoretical properties and good practical performance, the classical interior-point predictor–corrector strategy exhibits failures on certain problems, including relatively small systems. These failures are traced to a fundamental issue: the central path is not necessarily monotone with respect to the barrier parameter. As a result, enforcing a strictly decreasing barrier parameter—an implicit assumption of standard interior-point algorithms—may prevent convergence, even when a smooth solution path exists. This phenomenon is well known in numerical continuation and homotopy methods, but is rarely addressed explicitly in the context of frictional contact mechanics.

Key contribution: IPM-ANM coupling. To overcome this limitation, the main methodological contribution of this work is the integration of the Asymptotic Numerical Method (ANM) into the interior-point framework. Rather than advancing point-by-point along the central path, ANM constructs high-order local series expansions of the solution with respect to a continuation parameter. This allows the algorithm to capture entire branches of the central path, including regions where the parametrization with respect to the barrier parameter is non-monotone. The IPM-ANM approach reformulates the perturbed interior-point equations as a system with linear, bilinear, and constant terms, which is particularly well suited to ANM. At each continuation step, a sequence of linear systems—sharing the same Jacobian structure as the interior-point method—is solved to compute the coefficients of the asymptotic expansion. An estimate of the radius of convergence then determines the admissible continuation step. *Crucially, ANM removes the need for a strictly decreasing barrier parameter*, thereby restoring robustness in cases where classical Mehrotra-type algorithms fail. This represents a significant conceptual and practical advance for second-order solvers in frictional contact mechanics.

Numerical validation. The proposed methods are validated on a large benchmark set from the Frictional Contact Library (FCLIB)[2], including problems with up to several thousand contacts and highly redundant constraint configurations. The performance of the standard IPM and the enhanced IPM-ANM is compared against two widely used first-order methods: the Non-Smooth Gauss–Seidel (NSGS) method and the Alternating Direction Method of Multipliers (ADMM). The results demonstrate that:

- The interior-point method significantly outperforms first-order solvers in terms of accuracy and convergence rate on medium to large problems.
- Failures observed with the standard IPM are effectively mitigated by the ANM-based path-following strategy.
- IPM-ANM exhibits superior robustness on difficult, hyperstatic systems where both classical second-order and first-order methods struggle.

Conclusion. This work establishes *IPM-ANM as a robust and scalable second-order solver for frictional contact problems.* By explicitly addressing the non-monotone nature of the central path through asymptotic numerical continuation, the proposed approach bridges the gap between theoretical guarantees and practical robustness. The methodology opens new perspectives for the reliable simulation of complex contact-dominated mechanical systems, particularly in regimes where redundancy and nonsmoothness have traditionally limited the applicability of second-order methods.

References

[1] Vincent Acary, Paul Armand, Hoang Minh Nguyen, and Maksym Shpakovych. Second-order cone programming for frictional contact mechanics using interior point algorithm. *Optimization Methods and Software*, 39(3):634–663, 2024.

[2] Vincent Acary, Maurice Brémond, Tomasz Koziara, and Franck Pérignon. FCLIB: a collection of discrete 3D Frictional Contact problems. Technical Report RT-0444, INRIA, February 2014.

[3] Vincent Acary, Florent Cadoux, Claude Lemaréchal, and Jérôme Malick. A formulation of the linear discrete Coulomb friction problem via convex optimization. *ZAMM Z. Angew. Math. Mech.*, 91(2):155–175, 2011.

[4] Bruno Cochelin, Noureddine Damil, and Michel Potier-Ferry. The asymptotic-numerical method: an efficient perturbation technique for nonlinear structural mechanics. *Revue européenne des éléments finis*, 3(2):281–297, 1994.

Multiple pointwise contact search in engineering applications considering enforcement by a hybrid-barrier interface

Lucas da Silva and Alfredo Gay Neto

Escola Politécnica da Universidade de São Paulo, Brazil

A diverse set of engineering applications involves contact mechanics. For instance, in multibody dynamics (MBD), contact may appear as a constraint or an interaction term between bodies [1]. The same applies to particle systems treated with the Discrete Element Method (DEM) [2]. In the cited research, smooth surfaces are considered, the contacts are pointwise, and the pairs of contact points satisfy the common-normal condition. However, it is known that this condition is not sufficient to elect a valid pointwise contact pair [3]. For convex bodies, such a pair is unique and is more easily distinguishable among multiple critical pairs. This is not the case when contact surfaces are nonconvex, since multiple valid solutions may exist.

Gay Neto and Wriggers [4] introduced an interface including a barrier preventing interpenetration for the enforcement of contact between polyhedra. Considering the same interface for 2D contact of curved bodies, da Silva et al. [5] showed that pairs contributing to contact must, besides satisfying the common-normal condition, minimize the distance in their neighborhood. The same can be demonstrated for the 3D case. In the present work, we present discussions on a generalization for 3D of the proposal algorithm in [5], thus seeking strategies for determining multiple pointwise contact solutions in nonconvex scenarios. For that, ideas are illustrated in two examples in which these conditions are verified (Fig. 1). The first is the contact of two chain links that occur between the parts with negative Gaussian curvature. The second is an application in railway engineering, a free wheelset entering a curved track; the wheel profile is slightly concave, making the contact surface nonconvex.

(a) (b)

Figure 1: Contact of nonconvex geometries with more than one solution: (a) chain links, (b) wheel-rail interface.

References

[1] L.X. Xu, A method for modelling contact between circular and non-circular shapes with variable radii of curvature and its application in planar mechanical systems, *Multibody Syst. Dyn.*, 39:153-174, 2017.

[2] M.V. Craveiro, A. Gay Neto, and P. Wriggers, Contact between rigid convex NURBS particles based on computer graphics concepts, *Comput. Methods Appl. Mech. Engrg.*, 386:114097, 2021.

[3] D.S. Lopes, M.T. Silva, J.A. Ambrósio, and P. Flores, A mathematical framework for rigid contact detection between quadric and superquadric surfaces, *Multibody Syst. Dyn.*, 24:255-280, 2010.

[4] A. Gay Neto and P. Wriggers, Discrete element model for general polyhedra, *Computational Particle Mechanics*, 9:353-380, 2022.

[5] L. da Silva, M.V. Craveiro, and A. Gay Neto, Addressing concave boundaries in two-dimensional pointwise contact detection under the common-normal concept, *Comput. Methods Appl. Mech. Engrg.*, 438:117865, 2025.

Toward Multi-Scale Learning of Adhesive Contact Forces: From Single Asperities to Rough Surfaces

Ali Maghami[1], Merten Stender[2], Michele Ciavarella[1], and Antonio Papangelo[1]

[1] *Department of Mechanics, Mathematics, and Management, Politecnico di Bari, Bari, Italy*
[2] *Chair of Cyber-Physical Systems in Mechanical Engineering, Technische Universität Berlin, Germany*

Prediction of contact forces is essential for robotics, virtual prototyping, and physically based rendering of deformable objects. However, modeling viscoelastic adhesion and rough-surface multi-asperity interactions remains computationally expensive, limiting their use in real-time applications [1]. Based on our previous efforts [2], we present a data-driven framework that learns the full-time evolution of adhesive forces in viscoelastic Hertzian contacts and generalizes these predictions to multi-asperity and rough-surface settings. Our method begins with high-fidelity single-asperity simulations or experiments, from which we construct a coherent global time–displacement representation of viscoelastic indentation. We then train several stateful sequence-to-sequence neural networks (CNN-based and LSTM-based architectures) to predict the corresponding time-resolved adhesive force response. The models successfully learn rate effects, hysteresis, and unloading behavior directly from data, enabling fast and stable inference suitable for interactive applications. Figure 1a illustrates the single-asperity Hertzian model, and Figure 1b shows the learning performance of the four architectures evaluated. Finally, following proven concepts in multi-asperity modeling [3], we embed the learned single-asperity response into a multi-asperity formulation with a global reference surface. This framework lays the groundwork for real-time, physically accurate adhesive contact modeling in computer graphics and animation.

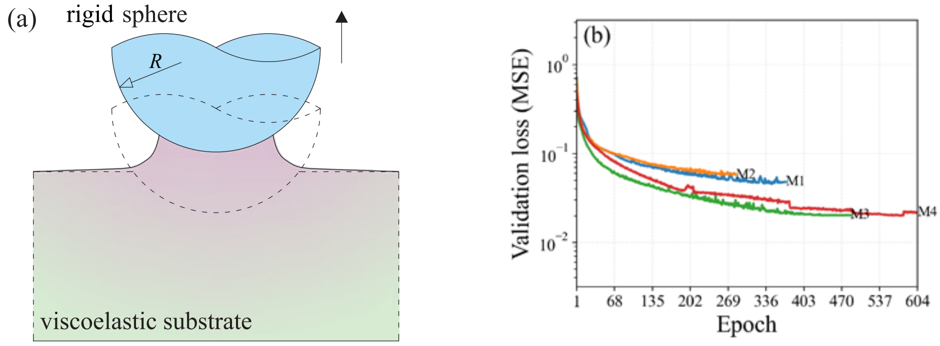


Figure 1: (a) Geometry of the viscoelastic Hertzian single-asperity model. (b) Learning curves for the four architectures evaluated (CNN-based, LSTM-based, and hybrids).

References

[1] A. Maghami, Q. Wang, M. Tricarico, M. Ciavarella, Q. Li, and A. Papangelo, Bulk and fracture process zone contribution to the rate-dependent adhesion amplification in viscoelastic broad-band materials, *J. Mech. Phys. Solids*, 193:105844, 2024.

[2] A. Maghami, M. Stender, and A. Papangelo, Pull-off force prediction in viscoelastic adhesive Hertzian contact by physics augmented machine learning, *Int. J. Solids Struct.*, 322:113584, 2025.

[3] A. Papangelo, N. Hoffmann, and M. Ciavarella, Load-separation curves for the contact of self-affine rough surfaces, *Sci. Rep.*, 7(1):6900, 2017.

Adhesion modelling in wheel-rail frictional contact

Andrzej Myśliński[1], Andrzej Chudzikiewicz[2]

[1]*Systems Research Institute, Warsaw, Poland, e-mail*: *myslinsk@ibspan.waw.pl*
[2]*Kazimierz Pulaski University of Technology and Humanities in Radom, Faculty of Transport, Radom, Poland, e-mail*: *ach1@wt.pw.edu.pl*

The paper is concerned with the analysis and numerical evaluation of adhesion and fatigue [1,2] in wheel-rail contact problems. Adhesion has a noticeable impact on both contact pressure distribution and contact length variation during each cycle [3,4].

The two-dimensional wheel-rail contact problem between a rigid wheel and an elasto-plastic rail lying on a rigid foundation is investigated. The contact phenomenon includes Coulomb friction, frictional heat generation as well as the wear of the contacting surfaces. The displacement and stress of the rail in contact are governed by the coupled elasto-plastic and heat conductive equations. The elastic and plastic responses are approximated, respectively, by Hooke's law and by von Mises yield criterion with isotropic power law hardening. The wear depth function appears as an internal variable in the non-penetration condition updating the gap between the worn surfaces of the bodies. Moreover the dissipated energy due to friction is calculated to evaluate the loss of the rail material and to determine the shape of the contacting surfaces during the wear evolution process. Therefore the wear phenomenon is modeled by the combined Archard and power dissipation models. The adhesion contact model is built based on RCC model [5]. The tangential resistance is based on the definition of the shear strength and the surface free energy [1,5].

The contact problem is solved numerically. The finite element method is used to discretize it. The original coupled problem is solved numerically using the splitting method [6]. In this approach first for a given temperature the displacements, stresses and wear depth are calculated using the generalized Newton method. The plastic flow and friction inequality conditions are reformulated as equality conditions using the nonlinear complementarity functions [7]. In the next step, for a given displacement and stress, the temperature is updated using Cholesky method. The numerical results, including the distribution of von-Mises stress, plastic strains, and the tangential traction, are discussed. .

References

[1] Q. Li, V.L. Popov, Non-adhesive and adhesive contacts of an elastic quarter-or eighth-space with freely sliding sides, *Friction*, 12:2052–2063, 2024.

[2] Y. Zhu, U. Olofsson, A. Söderberg, Adhesion modeling in the wheel-rail contact under dry and lubricated conditions using measured 3D surfaces, *Tribology International*, 61:1-10, 2013.

[3] J.C. Mergel, J. Scheibert, R.A. Sauer, Contact with coupled adhesion and friction: Computational framework, applications, and new insights, *Journal of the Mechanics and Physics of Solids*, 146, :104194, 2021.

[4] M. Shen, J. Li, L. Li, et al., Adhesion and Damage Behaviour of Wheel–Rail Rolling–Sliding Contact Suffering Intermittent Airflow with Different Humidities and Ambient Temperature, *Tribol. Lett* 72:18, 2024.

[5] L.B. Hu, Y. Cong, P. Joli, Z.Q. Feng, A bi-potential contact formulation for recoverable adhesion between soft bodies based on the RCC interface model, *Comput. Methods Appl. Mech. Engrg.* 390:114478, 2022.

[6] A. Myśliński, A. Chudzikiewicz, Wear modelling in wheel-rail contact problems based on energy dissipation, *Tribology - Materials, Surfaces & Interfaces*, 15:138-149, 2021.

[7] C. Hager, B.I. Wohlmuth, Nonlinear complementarity functions for plasticity problems with frictional contact, *Comput. Methods Appl. Mech. Engrg*., 198: 3411-3427, 2009.

Recent advancement in understanding vibroadhesion: experiments and modelling

A. Papangelo, M. Tricarico, M. Ciavarella

TriboDynamics Lab, Department of Mechanics, Mathematics and Management, Politecnico di Bari, via Orabona 4, Bari, 70125, Italy

Modulation of interfacial adhesion in soft materials is of growing importance for engineering systems such as soft robotic grippers, programmable manipulators, and reversible adhesive interfaces [1]. A promising emerging approach, referred to as vibro-adhesion [2,3], exploits fast (>100 Hz), low-amplitude (<100 µm) substrate vibrations to rapidly tune interfacial toughness. Although vibrations are known to influence frictional and contact phenomena, their role in adhesion of soft viscoelastic materials has remained comparatively unexplored. This work presents an integrated experimental–theoretical investigation that reveals how high-frequency excitations can dramatically strengthen or weaken soft adhesive contacts through viscoelastic dissipation mechanisms. The study focuses on the axisymmetric contact between a rigid borosilicate spherical lens and a soft PDMS (polydimethylsiloxane) substrate. The lens is hang on a compliant spring while the nominally flat PDMS substrate is harmonically excited using a pair of electrodynamic shakers [3]. Under excitation the contact patch behaves as a dynamically evolving external crack, in which the processes of crack propagation and healing are strongly rate dependent. Because of viscoelastic energy losses the maximum adherence force (pull-off) can be increased by about 15 times with respect to the quasi-static unloading case without vibrations [3]. A reduced-order model is developed that couples a viscoelastic adhesive contact formulation with the nonlinear dynamics of the suspended spherical indenter. The model captures how vibration-induced oscillations of the contact radius interact with the system's resonance behavior [3,4,5]. Near resonance, the contact edge undergoes rapid cyclic motion, amplifying viscoelastic dissipation within the substrate bulk and dramatically increasing adhesion. The framework enables prediction of the dependence of pull-off force on excitation frequency and amplitude, and demonstrates that adhesion regulation is fundamentally linked to the interplay between material rheology and oscillator dynamics. Comparison between model predictions and experimental measurements shows excellent agreement across a broad operating range although deviations emerge for vibration amplitudes exceeding approximately 100 µm, suggesting the onset of nonlinear effects leading to contact instabilities [3,4,5]. The effects of material properties, preload, substrate thickness, unloading rate is also investigated, showing a rich behavior which could be exploited for the development of smart interfaces capable of actively tune adhesion for robotics applications.

References

[1] Giordano, G., Scharff, R. B. N., Carlotti, M., Gagliardi, M., Filippeschi, C., Mondini, A., ... & Mazzolai, B. (2024). Mechanochromic Suction Cups for Local Stress Detection in Soft Robotics. *Advanced Intelligent Systems*, 2400254.

[2] Shui, L., Jia, L., Li, H., Guo, J., Guo, Z., Liu, Y., ... & Chen, X. (2020). Rapid and continuous regulating adhesion strength by mechanical micro-vibration. *Nature communications*, *11*(1), 1583.

[3] Tricarico, M., Ciavarella, M., & Papangelo, A. (2025). Enhancement of adhesion strength through microvibrations: Modeling and experiments. *Journal of the Mechanics and Physics of Solids*, *196*, 106020.

[4] Ciavarella, M., Tricarico, M., & Papangelo, A. (2025). On the dynamic JKR adhesion problem. *Mechanics of Materials*, *202*, 105252.

[5] Argatov, I., Papangelo, A., & Ciavarella, M. (2025). An asymptotic model of vibroadhesion. *International Journal of Non-Linear Mechanics*, *174*, 105089.

Physical origin of friction relaxation after interrupted sliding

Baptiste Boulet, D. Dalmas, J. Scheibert

CNRS, Ecole Centrale de Lyon, ENTPE, LTDS, UMR5513, 69130 Ecully, France

One of the most generic friction laws is the so-called rate-and-state friction law [1]. It combines the time-evolution of the static friction coefficient and the sliding-velocity-dependence of the dynamic friction coefficient. It is parametrized by two empirical parameters, which can be calibrated using macroscopic tests: the slide-hold-slide and the velocity step tests.

In the slide-hold-slide test, the friction coefficient decreases during hold (when the contact is left at rest after full sliding is interrupted). However, the physical origin of this relaxation remains unclear: is it due to bulk relaxation, interfacial slip-based relaxation or a combination of both? In this work, we address this fundamental question for dry elastomer/glass contacts. Our strategy is to monitor experimentally the displacement field at the contact interface when sliding is stopped, using digital image correlation techniques. This allows us to quantify the amount of slip occurring after the motion cessation.

We conducted experiments on an elastomer sphere in contact with a glass plane during interrupted shear tests at various fractions of the shearing distance required to obtain full-slip of the interface. The sphere is made, for optical purposes, of polydimethylsiloxane (PDMS), and has a curvature radius of R=9.42 mm. To enable in operando evaluation of the shear displacement field thanks to digital image correlation (DIC) analysis [2], a thin subsurface layer of particles is embedded as close as possible to the PDMS sphere surface. The resulting displacement fields provide insights into the local behavior at the contact interface, particularly in identifying sticking and slipping zones [3] during post-shearing relaxation. We quantified the time-dependence evolution of the slip field, as a function of the normal force and of the shear distance before stopping, not only under standard frictional conditions, but also in fully stuck conditions by preventing any contact slipping during the shear.

Based on those results, we will discuss the combination of mechanisms involved in post-shear relaxation, shedding fundamental insights into the origin and contact history dependence of the static friction force.

References

[1] T. Baumberger, and C. Caroli, (2006). Solid friction from stick–slip down to pinning and aging. *Advances in Physics*, 55(3–4), 279–348.

[2] A. Prevost, J. Scheibert and G. Debrégeas, Probing the micromechanics of a multi-contact interface at the onset of frictional sliding. *Eur. Phys*. J. E 36, 17 (2013).

[3] J. Lengiewicz, M. de Souza, M.A. Lahmar, C. Courbon, D. Dalmas, S. Stupkiewicz, J. Scheibert. Finite deformations govern the anisotropic shear-induced area reduction of soft elastic contacts. *Journal of the Mechanics and Physics of Solids* 143, 104056 (2020).

Shifted–Penalty Multigrid Method for Contact

G. Marchi[2], H. Kothari[1,2], R. Krause[1,3], A. Nelson[4], P. Vassilevski[4] and P. Zulian[1,2]

1. UniDistance Suisse - Switzerland, 2. Euler Institue USI - Switzerland, 3. KAUST - Saudi Arabia, 4. Portland State University - Oregon USA.

High-performance computing is essential for efficiently solving large-scale contact problems. Simulating such phenomena at engineering scale is often limited by computational resources, making it crucial to design algorithms that fully exploit modern hardware like multi-core CPUs and GPUs. Iterative solvers and preconditioners play a central role in this efficiency.

Monotone Multigrid (MMG) methods offer optimal complexity and robustness. In parallel, Penalty and Augmented Lagrangian methods handle over-constrained and fuzzy constraints effectively. Among these, the Shifted-Penalty method is notable for accurately enforcing constraints while remaining competitive with non-smooth techniques like the semi-smooth Newton method.

To combine the optimal complexity of MMG with the flexibility of shifted-penalty methods, we introduce the Shifted-Penalty Multigrid (SPMG) method. Designed from the ground up for GPU architectures, SPMG unifies nonlinear smoothing with constraint-aware multigrid strategies.

Our implementation uses matrix-free differential operators and memory-efficient semi-structured meshes to discretize elasticity equations. We present the SPMG algorithm with a focus on nonlinear smoothing and constraint coarsening.

We evaluate performance on the Grace-Hopper superchip of the CSCS Alps supercomputer. Emphasis is placed on single-node GPU performance, kernel design, and convergence behavior in simple contact scenarios. Finally, we demonstrate SPMG's scalability on large-scale problems with hundreds of millions of degrees of freedom.

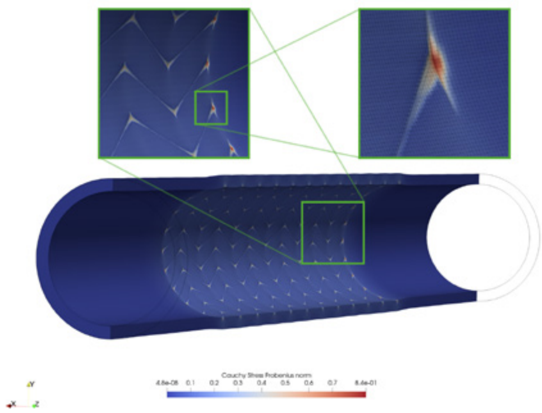


Figure 1: High resolution stress computation for nonlinear contact problem - 268 319 520 dofs on Apple M1 Max

References

[1] G. Zavarise, The shifted penalty method, *Computational Mechanics.*, 56:1–17, 2015.

[2] G. Marchi, H. Kothari, R. Krause, A. Nelson, P. Vassilevski and P. Zulian, Shifted–Penalty Multigrid Method for Contact, *In preparation.*

Application of the Nitsche method to three industrial 3D elastoplastic frictional contact problems

C. Pozzolini[(1)], Y. Renard[(2)], M. Pignol[(1)], B. Souloumiac[(1)]

[(1)]*Framatome, Zone Tech, 2 Rue Professeur Jean Bernard, 69007 Lyon. France*
cedric.pozzolini@framatome.com,moise.pignol@framatome.com,bruno.souloumiac@framatome.com,
[(2)]*INSA LYON, ICJ UMR5208, LaMCoS UMR5259, F-69621, Villeurbanne, France*
Yves.Renard@insa-lyon.fr

In recent decades, Nitsche's method has emerged as a powerful and elegant alternative for the numerical treatment of contact between deformable solids. Originally introduced in 1971 by Nitsche [1] for the weak imposition of boundary conditions, it has since been extensively applied to interface-coupled problems. Its first application to contact mechanics was presented in 2008 by Wriggers et al. [2], but it was an non-consistent formulation. Another complete mathematical analysis of linearized elasticity provided las early as 2013 by Chouly, Hild, Renard et al. unlocked the problem with a new consitent formulation [3,4,5]. The principal advantage of Nitsche's method is that it is variationally consistent, ensuring optimal convergence rates, without introducing additional degrees of freedom like the Lagrange multiplier method. This eliminates the need to satisfy a discrete inf-sup condition. This advantage comes at the cost of having to evaluate the boundary traction from the continuum stresses. To enhance stability, advanced symmetric and skew-symmetric variants have been developed, the latter being stable for any positive penalty parameter. The method has been successfully extended to more complex scenarios, including finite deformation elasto-plastic contact. In this context, the approximation strategy proposed here, which implements a weak integral contact condition conceptually similar to Lagrange multipliers but within the Nitsche framework, was first developed and validated in the open-source finite element library GetFEM [6,7]. It has been applied to a wide range of contact scenarios, including small and large deformations for elastic and hyperelastic materials, with and without friction.The goal of this note is to describe the application of Nitsche's method to enforce contact conditions, with or without Coulomb friction, between two elasto-plastic bodies. after that, we present how this method has been operationalized within the industrial finite element software SYSTUS/SYSWELD 2024 to solve practical engineering problems, demonstrating its effectiveness within the small deformations' framework. In this work, we employ segment-to-segment integration on each slave element and enforce contact constraints at each slave element. and the contact detection is done at Gaussian points, which is very advantageous.

References

[1] J. Nitsche. *Uber ein Variationsprinzip zur Losung von Dirichlet-Problemen bei Verwendung von Teilraumen, die keinen Randbedingungen unterworfen sind.*, Abhandlungen aus dem Mathematischen Seminar der Universitat Hamburg, 36:1(1971), pp. 9--15.

[2] P. Wriggers and G. Zavarise,A *formulation for frictionless contact problems using a weak form introduced by Nitsche,* Comput. Mech., 41, 2008.

[3] F. Chouly, M. Fabre, P. Hild, J. Pousin, Y. Renard, *An overview of recent results on Nitsche's method for contact problems*, Proceedings of the UCL Workshop 2016 on Geometrically Unfitted Finite Element Methods and Applications in Lect. Notes Comput. Sci. Eng, Vol. 121, Springer, 2017.

[4] F. Chouly, R. Mlika, Y. Renard. *An unbiased Nitsche's formulation of large deformation frictional contact and self-contact,* Comp. Meth. Appl. Mech. Engng., 2017.

[5] F. Chouly and P. Hild. *A Nitsche-based method for unilateral contact problems: numerical analysis. SIAM J. Numer. Anal.*, 51:2(2013), pp. 1295--1307.

[6] J. Pommier, Y. Renard. GetFEM++, an open source generic C++ library for finite element methods. http://download.gna.org/getfem/html/homepage/.

[7] Y. Renard, K. Poulios. GetFEM: *Automated FE modeling of multiphysics problems based on a generic weak form language*, Transactions on Mathematical Software, 47:1:1--31, 2020.

Geometric Potential and Geometric Predicates for Unconditionally Robust Elastodynamics Simulation

Daniele Panozzo
New York University

Abstract

The numerical solution of partial differential equations (PDE) is ubiquitously used for physical simulation in scientific computing and engineering. Ideally, a PDE solver should be opaque: the user provides as input the domain boundary, boundary conditions, and the governing equations, and the code returns an evaluator that can compute the value of the solution at any point of the input domain. This is surprisingly far from being the case for all existing open-source or commercial software, despite the research efforts in this direction and the large academic and industrial interest. To a large extent, this is due to lack of robustness in geometric algorithms used to create the discretization, detect collisions, and evaluate element validity.

I will present an interior point simulation approach, which provides strong robustness guarantees in simulation codes, ensuring, for the first time, validity of the trajectories accounting for floating point rounding errors over an entire elastodynamic simulation with contact, building on the Incremental Potential Contact (IPC) framework [6]. The algorithm assumes a valid initial state void of intersections or inverted elements, and guarantees that these properties will hold for the entire trajectory while accounting for floating point rounding error. The two major building blocks of this approach are the conversion of time stepping into an unconstrained minimization of a set of continuous potentials (modeling elastic, contact, and friction forces), and a robust, conservative line-search to check for collisions between geometric primitives and for ensuring validity of the deforming elements over linear trajectories.

I will introduce the general framework first [6], then dive into the definition of a smooth geometric contact potential for barrier-based contact handling [3], and finally introduce algorithms to automatically construct robust geometric predicates for collisions and inversions from a high-level specification [8, 7]. The overall system enables simulations of scenes with unprecedented geometric and contact complexity which appear in structural mechanics, microscopy, and biomechanics, including traction force estimation on a live zebrafish and efficient modeling and simulation of fibrous materials. The formulation is a naturally differentiable solver [5], opening the door to exciting geometric optimization problems such as the discovery of optimal shock-absorbing microstructures for elastic metamaterials [4], robotics grippers optimized via soft pneumatic actuator design [1], and deformable sensors based on capacitive stretch sensing [2].

Bio

Daniele Panozzo is a Full Professor of Computer Science and Vice-Dean of Academic Affairs at the Courant Institute School of Mathematics, Computing, and Data Science. Prior to joining NYU he was a postdoctoral researcher at ETH Zurich (2012–2015). Daniele earned his PhD in Computer Science from the University of Genova (2012) and his doctoral thesis received the EUROGRAPHICS Award for Best PhD Thesis (2013). He received the EUROGRAPHICS Young Researcher Award in 2015, the NSF CAREER Award in 2017, and a Sloan Research Fellowship in 2020. Daniele's research group is leading the development of PolyFEM (`https://polyfem.github.io`), a simple and robust finite element library, and wild meshing (`https://github.com/wildmeshing`), a 2D and 3D robust meshing library. Daniele initiated the Graphics Replicability Stamp (`http://www.replicabilitystamp.org`), which is an initiative to promote reproducibility of research results and to allow scientists and practitioners to immediately benefit from state-of-the-art research results. His research interests are in finite element simulation, digital fabrication, and geometry processing.

References

[1] Arvi Gjoka, Espen Knoop, Moritz Bächer, Denis Zorin, and Daniele Panozzo. Soft pneumatic actuator design using differentiable simulation. In *SIGGRAPH '24: Special Interest Group on Computer Graphics and Interactive Techniques Conference*, pages 1–11, New York, NY, USA, 2024. Association for Computing Machinery.

[2] Arvi Gjoka, Yongkang Sun, Roi Poranne, and Daniele Panozzo. Computational modeling and design of capacitive stretch sensors. *ACM Transactions on Graphics*, 44(6):240:1–240:15, 2025.

[3] Zizhou Huang, Max Paik, Zachary Ferguson, Daniele Panozzo, and Denis Zorin. Geometric contact potential. *ACM Transactions on Graphics*, 44(4), 2025.

[4] Zizhou Huang, Daniele Panozzo, and Denis Zorin. Optimized shock-protecting microstructures. *ACM Transactions on Graphics*, 43(6), 2024.

[5] Zizhou Huang, Davi Colli Tozoni, Arvi Gjoka, Zachary Ferguson, Teseo Schneider, Daniele Panozzo, and Denis Zorin. Differentiable solver for time-dependent deformation problems with contact. *ACM Transactions on Graphics*, 43(3), 2024.

[6] Minchen Li, Zachary Ferguson, Teseo Schneider, Timothy R. Langlois, Denis Zorin, Daniele Panozzo, Chenfanfu Jiang, and Danny M. Kaufman. Incremental potential contact: Intersection- and inversion-free large deformation dynamics. *ACM Transactions on Graphics*, 39(4), 2020.

[7] Federico Sichetti, Zizhou Huang, Marco Attene, Denis Zorin, Enrico Puppo, and Daniele Panozzo. High-order continuous geometrical validity. *ACM Transactions on Graphics*, 44(4), 2025.

[8] Federico Sichetti, Enrico Puppo, Zizhou Huang, Marco Attene, Denis Zorin, and Daniele Panozzo. MiSo: A DSL for robust and efficient solve and minimize problems. *ACM Transactions on Graphics*, 44(4), 2025.

Energy-Based Physics-Informed Neural Networks for Nonlinear Contact Mechanics

Daniel Wolff[1], Simon Völkl[1], Tarik Sahin[1], and Alexander Popp[1]

[1] *Institute for Mathematics and Computer-based Simulation, University of the Bundeswehr Munich*

The reliable and efficient simulation of contact problems remains a subject of interest in computational mechanics due to their inherent non-smoothness and strong nonlinearities. While established approaches solve such problems with classical numerical methods such as the Finite Element Method (FEM), we approach this challenge by applying physics-informed machine learning techniques, specifically energy-based physics-informed neural networks (EPINNs). The overarching goal is to develop machine learning surrogates that are not only physically consistent but also fast to evaluate, thereby enabling their use in multi-query settings such as optimization, uncertainty quantification, or inverse analysis; applications which are often computationally intractable when relying solely on conventional high-fidelity models.

Although classical data-driven approaches have demonstrated promising results in several engineering contexts, their reliance on extensive and high-quality datasets poses a fundamental limitation. In many practical scenarios, generating such data via FEM simulations is prohibitively expensive and experimental data acquisition may be infeasible. Physics-based machine learning attempts to overcome this issue by reducing the dependence on training data through the incorporation of physics-based regularization into the training process. Therefore, the recently proposed EPINN framework [1] formulates the learning task in an energy-minimization setting and trains a neural network to directly minimize the total potential energy functional.

In this contribution, we extend the EPINN framework for frictionless contact mechanics by introducing an incremental solution strategy that enables the network to recover the entire nonlinear load path rather than only the final equilibrium configuration. This is achieved by decomposing the full nonlinear optimization problem into a series of optimization problems with weaker nonlinearities. While current approaches employ a simple yet effective penalty potential to impose contact constraints [2], we also investigate an Augmented Lagrangian approach and compare both methods. Our numerical results will be quantitatively validated against high-fidelity FEM reference solutions to ensure accuracy and stress that this ML-based approach is also comparable to established methods in terms of performance.

References

[1] E. Samaniego, C. Anitescu, S. Goswami, V.M. Nguyen-Thanh, H. Guo, K. Hamdia, X. Zhuang, and T. Rabczuk. An energy approach to the solution of partial differential equations in computational mechanics via machine learning: Concepts, implementation and applications. *Comput. Methods Appl. Mech. Eng.*, 362:112790, 2020.

[2] Jinshuai Bai, Zhongya Lin, Yizheng Wang, Jiancong Wen, Yinghua Liu, Timon Rabczuk, YuanTong Gu, and Xi-Qiao Feng. Energy-based physics-informed neural network for frictionless contact problems under large deformation. *Comput. Methods Appl. Mech. Eng.*, 437:117787, 2025.

Advancing Parameter Extraction in Adhesive Contact Mechanics: A Method for Estimating the Tabor Parameter Across Adhesive Regimes

D. Dalmas and J. Scheibert

CNRS, Ecole Centrale de Lyon, ENTPE, LTDS, UMR 5513, Ecully,69130, France.

The accurate extraction of material parameters from contact experiments is a cornerstone of adhesive contact mechanics, enabling the quantitative comparison between theoretical models and experimental data. Among these parameters, the Tabor parameter (λ) is particularly critical, as it governs the transition between adhesion models, from DMT-like ($\lambda \to 0$) to JKR-like ($\lambda \to +\infty$) behaviors (Figure). However, traditional fitting methods often fail to capture the full spectrum of λ.

In this work, we identify an efficient fitting methodology to extract material parameters from sphere-plane contact data Our approach leverages the Maugis-Dugdale [1] model but is performed on normalized data, following the normalization proposed by Chaudhury et al. [2] (panel b of Figure). To validate its performance, we generated theoretical datasets simulating the evolution of contact area vs normal force for various combinations of sphere radius, Young's moduli and adhesion energies.

Our results demonstrate that our method outperforms established literature-proposed methods (DMT, JKR, Carpick et al. [3] or Chaudhury et al. [2]). The method's reliability is further confirmed under more experimentally realistic conditions by adding measurement noise. A key insight from this study is the importance of collecting data at the smallest possible contact radii, as these measurements are critical for precise λ estimation.

This work not only advances the accuracy of parameter extraction in contact mechanics but also highlights the necessity of careful data acquisition. By providing a more versatile and precise tool for parameter estimation, our method contributes to refining predictive models in tribology and contact mechanics.

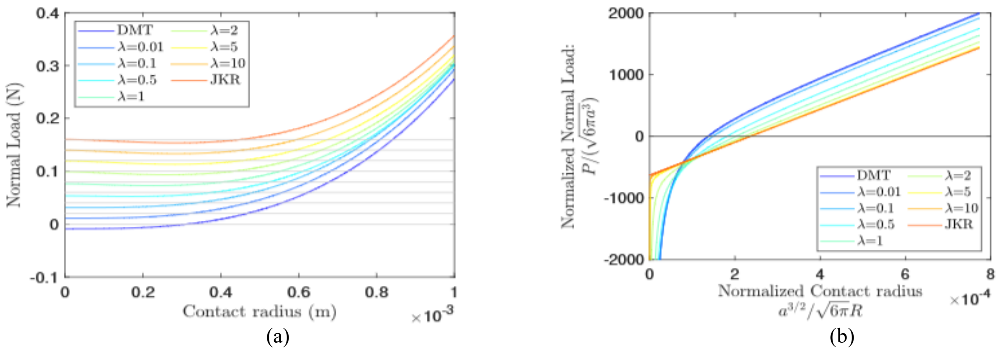


Figure 1: (a) Normal load as a function of the contact radius for DMT model, MD model with different λ ranging from 0.01 to 10 and JKR. Each curve is vertically offset by 0.02 for clarity. The system parameters are typical of a centimetric soft elastomer sphere. (b) The same data plotted using the normalization proposed by Chaudhury et al. [2].

References

[1] Maugis, D., *Adhesion of spheres: The JKR-DMT transition using a dugdale model*, 150(1), 243–269, 1992, https://doi.org/10.1016/0021-9797(92)90285-T

[2] Chaudhury, M.K., Weaver, T., Hui, C.Y., Kramer, E., *Adhesive contact ofcylindrical lens and a flat sheet* 80(1), 30–37, 1996, https://doi.org/10.1063/1.362819.

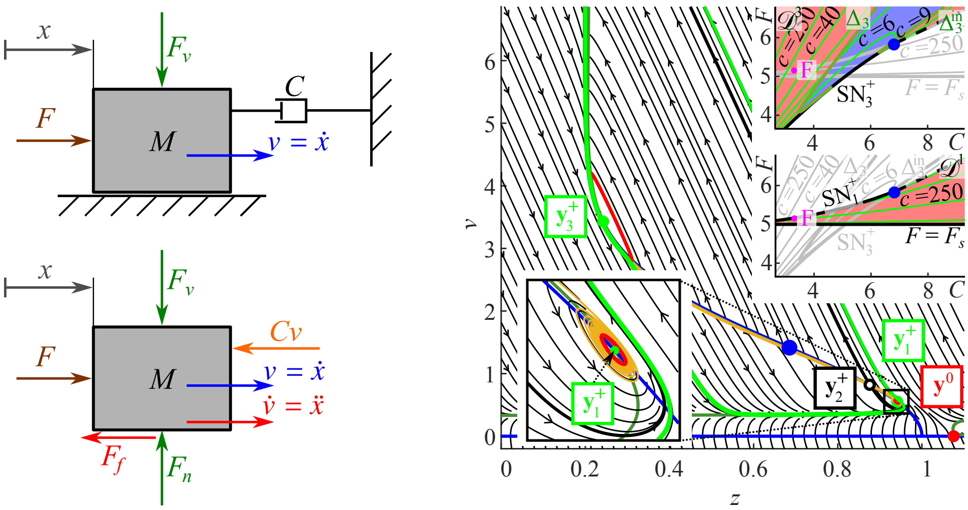
[3] Carpick, R.W., Ogletree, D.F., Salmeron, M, *A general equation for fitting con-tact area and friction vs load measurements*, 211(2), 395–400, 1999, https://doi.org/10.1006/jcis.1998.6027.

Monostable, bistable, and tristable scenarios in a simple mechanical model with LuGre friction

Balázs J. Békési, Gábor Csernák

Department of Applied Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, Műegyetem rkp. 3., H-1111 Budapest, Hungary,
bekesi@mm.bme.hu, csernak@mm.bme.hu

The frictional interaction between contacting bodies is based on very complex physical processes. To model their combined effect, researchers in the related fields have introduced numerous empirical models that are adapted to the needs of various application areas [1]. The present paper focuses on the widely used LuGre model. We consider a remarkably simple mechanical model: a block on a horizontal surface subjected to an external force F, the LuGre friction force F_f (including bristle damping) and a viscous damping force Cv (see Figure 1(a)).



(a) (b)

Figure 1: (a) Mechanical model and (b) a tristable scenario with two stable slipping equilibria ($\mathbf{y}_1^+$, $\mathbf{y}_3^+$) and a stable limit cycle. Stable/unstable solutions are drawn in green/red; v and z denote the velocity of the block and the internal variable of the LuGre model, respectively.

Following the analysis initiated in [2, 3], we point out several local and global bifurcations in the system that lead to the appearance of coexisting stable solutions in the phase-space, as shown in Figure 1(b). The implications of these results are discussed in our presentation in connection with the passivity property of the friction model.

References

[1] F. Marques, P. Flores, J.C. Pimenta Claro, H.M. Lankarani, A survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems, *Nonlinear Dyn* 86:1407–1443, 2016.

[2] B.J. Bekesi, M. Antali and G. Csernak, Phase portraits and bifurcations induced by static and dynamic friction models, *Nonlinear Dyn*, 113:15863–15899, 2025.

[3] B.J. Bekesi, G. Csernak, Self-excited fluctuation of sliding velocity induced by LuGre friction in a minimal mechanical model, *Int. J. Solids & Struct*, 315:113293, 2025.

A variational energy-based Boundary Element approach for 3D adhesive contacts

M. Santeramo[1], C. Putignano[1], S. Krenn[2], and G. Carbone[1]

[1] *Department of Mechanics, Mathematics and Management, Politecnico di Bari, Via Orabona 4, 70100, Bari, Italy.*

[2]*AC2T research GmbH, Viktor-Kaplan-Straße 2/C, 2700 Wiener Neustadt, Austria.*

The interplay between surface roughness and adhesion plays a central role in both industry and biology. Indeed, it is well established how adhesive effects can be highly reduced, or even neglected, by surface roughness. Hence, this fundamental relation has gathered the interest of the scientific community worldwide, thus leading to analytical theories for smooth contacts [1,2], together with numerical and experimental analyses [3,4]. However, the numerical procedures proposed based on the definition of an *ad-hoc* potential modelling the adhesive interaction: this assumption clearly introduces a strong dependance of the numerical predictions on the set of parameters selected to shape the adhesive potential, so that a very small change in one of these would lead to dramatic change in the adhesive interaction between the contacting solids.

Hence, we present an innovative variational energy-based Boundary Element methodology to investigate 3D contacts when adhesive effects are considered at the contact interface. Specifically, the methodology requires the total free energy to be minimized for a fixed value of penetration and, crucially, does not ask for a specific adhesive potential. Indeed, the adhesive interaction is fully characterized by the work of adhesion $\Delta\gamma$, which can be determined experimentally. Furthermore, the optimization procedure is served by a proper topographical description of the contact area patches, thus providing a more efficient and numerically time-saving optimization strategy, aiming at determining the real contact area corresponding to a minimum in the total free energy. To this regard, the solution up to a desired value of penetration is retrieved by moving with very small increments of indentation from a non-contact condition and minimizing the total energy at each step.

Ultimately, it is important to stress that the proposed methodology is not restricted to smooth contacts, as it can be deployed to assess rough contact problems and, most importantly, it is not limited to the adhesive case. This flexibility blazes the way to further studies where different surface energy contributions are governing the problem.

References

[1] Johnson, K.L., Kendall, K., Roberts, A., 1971. Surface energy and the contact of elastic solids. Proc. R. Soc. A 324 (1558), 301–313.

[2] B. N. J. Persson, O. Albohr, U. Tartaglino, A. I. Volokitin, E. Tosatti, On the nature of surface roughness with application to contact mechanics, sealing, rubber friction and adhesion, Journal of Physics: Condensed Matter 17 (1) (2004) R1.

[3] C. Campañá, M. H. Müser, Contact mechanics of real vs. randomly rough surfaces: A green's function molecular dynamics study, Europhysics Letters 77 (3) (2007) 38005.

[4] Bennett, A.I., Harris, K.L., Schulze, K.D., Urueña, J.M., McGhee, A.J., Pitenis, A.A., Müser, M.H., Angelini, T.E., Sawyer, W.G., 2017. Contact measurements of randomly rough surfaces. Tribol. Lett. 65 (4)

Liquid Hertz impact

Ivan Argatov and Vitaly Kocherbitov

[1] *Department of Biomedical Science, Malmö University, Jan Waldenströms gata 25, Malmö, SE-205 06, Sweden*

[2] *Biofilms – Research Center for Biointerfaces, Malmö University, Per Albin Hanssons väg 35, Malmö, SE-205 06, Sweden*

The interaction between a drop of liquid and a rigid substrate upon collision depends on many physical factors, including the drop surface tension, γ; inertial properties (characterized by the drop density, ρ); external force fields (for example, gravity); and the wetting properties of the surface. Impact parameters such as the drop size (characterized by the drop's equivalent radius, R) and the impact velocity, v_0, govern the transition between different impact regimes. The Weber number, $\mathrm{We} = \rho R v_0^2/\gamma$, which represents the ratio of inertial to surface tension forces, naturally arises in the impact problem and serves as the key dimensionless group.

Recently, Gabbard *et al.* [1] reviewed the state of the art in analytical modeling of drops impacting non-wetting substrates at low Weber numbers. Within the landscape of mathematical models, two are particularly prominent: the model by Chevy *et al.* [2] and that of Moláček and Bush [3]. The present study further develops the first approach by relaxing its assumption of small drop deformations. Specifically, we construct a quasi-static model for the normal impact of a viscous drop on a non-wetting substrate. The axisymmetric deformation of a sessile drop is described analytically using new asymptotically exact approximations to solutions of the Young–Laplace equation. Viscous dissipation is accounted for in linearized form through a damping coefficient inversely proportional to the relaxation time of small-amplitude oscillations of a viscous sessile drop. This formulation enables evaluation of the key characteristics of Hertz-type impact at low Weber numbers, including the drop spreading factor, restitution coefficient, and characteristic time scale. Comparison with experimental data demonstrates that the model reliably captures the essential features of slow, viscously-damped liquid-drop impacts on non-wetting surfaces.

References

[1] C.T. Gabbard, E.A. Agüero, R. Cimpeanu, K. Kuehr, E. Silver, J.-W. Barotta, C.A. Galeano-Rios, and D.M. Harris, Drop rebound at low Weber number, *J. Fluid Mech.*, 1019:A25, 2025.

[2] F. Chevy, A. Chepelianskii, D. Quéré, and E. Raphaël, Liquid Hertz contact: softness of weakly deformed drops on non-wetting substrates, *Europhys. Lett.*, 100:54002, 2012.

[3] J. Moláček and J.M. Bush, A quasi-static model of drop impact, *Phys. Fluids*, 24:127103, 2012.

Mixed-dimensional beam-to-solid interaction: From embedded fibers to contact

Ivo Steinbrecher, Alexander Popp

Institute for Mathematics and Computer-Based Simulation,
University of the Bundeswehr Munich (UniBw M), Germany,
www.unibw.de/imcs-en ivo.steinbrecher@unibw.de

Interactions between slender fiber- or rod-like components and three-dimensional solids are central to many mechanical systems in engineering and bio-mechanics. Numerical models typically rely on the finite element method and often require a trade-off between accuracy and computational cost. For fiber-reinforced materials, two common strategies exist: (i) homogenization, where fiber and matrix stiffnesses are combined into an anisotropic material law, reducing meshing effort but limiting insight into local fiber-matrix interactions; and (ii) fully resolved 3D modeling, which captures detailed behavior but is computationally expensive. The main idea of the presented framework is to provide finite element formulations to combine the successful and rich history of nonlinear beam theories based on 1D-Cosserat continua with classical 3D continuum finite elements. This allows to explicitly model the fibers while still maintaining a moderate overall model complexity compared to the fully resolved model. The resulting problems are referred to as beam-to-solid (BTS) interaction problems.

Because beam and solid equations differ in dimensionality, BTS problems are inherently mixed-dimensional. Coupling a 1D beam to a 3D solid introduces a line-load-type singularity, whose influence on the global response is negligible as long as the beam cross-section is smaller than the solid elements. A second assumption is that fiber volume is not removed from the matrix, producing overlapping domains. For typical fiber-matrix stiffness ratios, this inconsistency is negligible and greatly simplifies meshing, allowing independent discretizations and non-matching grids.

The BTS framework can be applied to various beam and solid formulations. Interaction constraints are enforced along the beam centerline using a weighted mortar approach with penalty regularization, eliminating Lagrange multipliers and yielding robust coupling. Special attention is required when coupling rotational degrees of freedom, which demands a suitable triad field within the solid. For beam-surface interactions, the surface normal also enters the constraint formulation. This can include problems where the beam is coupled to the solid surface and problems where the beam is in unilateral contact with the solid surface.

Among the main topics addressed in this talk are a detailed outline of the beam-to-solid interaction framework, as well as a discussion of the characteristic traits of mixed-dimensional interaction in solid mechanics. Selected quantitative and qualitative examples are presented to highlight in order to underline the usability for real life science and engineering applications

References

[1] C. Meier, A. Popp and W.A. Wall, Geometrically exact finite element formulations for slender beams: Kirchhoff–Love theory versus Simo–Reissner theory, *Archives of Computational Methods in Engineering*, 26, 163–243, 2019.

[2] I. Steinbrecher, M. Mayr, M.J. Grill, J. Kremheller, C. Meier and A. Popp, A mortar-type finite element approach for embedding 1D beams into 3D solid volumes, *Computational Mechanics*, 66, 1377–1398 , 2020.

[3] I. Steinbrecher, A. Popp and C. Meier, Consistent coupling of positions and rotations for embedding 1D Cosserat beams into 3D solid volumes, *Computational Mechanics*, 69, 701–732, 2022

[4] I. Steinbrecher, N. Hagmeyer, C. Meier and A. Popp, A Consistent Mixed-Dimensional Coupling Approach for 1D Cosserat Beams and 2D Surfaces in 3D Space, *Computational Mechanics*, 76, 1233–1260, 2025.

An optimization-based framework to design metainterfaces with tailored friction faws

L. Fu, D. Gabriel Kashala, D. Dalmas and J. Scheibert

CNRS, Ecole Centrale de Lyon, ENTPE, LTDS, UMR5513, 69130 Ecully, France.

Providing dry solid contacts with on-demand macroscale frictional behaviour remains a grail in tribology, haptics or robotics. Metainterfaces created from surfaces with engineered asperity-based topographies can achieve such friction control. However, only few friction behaviours were demonstrated because suitable topographies were identified based on human intuition [1]. Here, I will introduce a numerical-optimisation-based inverse design framework to automatically discover new metainterfaces satisfying specified relationships between friction and normal forces (friction law). To illustrate the framework's versatility, I will show how to (i) expand the range of achievable friction coefficients at a constant material pair (ii) unlock power-law friction laws with arbitrary exponents between 2/3 and 1, and (iii) achieve bilinear laws with a smaller slope in the second segment than in the first. Relevant cases will be validated experimentally. By systematically exploring large parameter spaces, not limited to topography but potentially incorporating the individual asperities' bulk material or surface physicochemistry, this automated framework offers design solutions for any physically possible friction law. It also provides new insights into the elusive relationship between local interfacial properties and macroscopic friction [2].

References

[1] A. Aymard, E. Delplanque, D. Dalmas, J. Scheibert, Designing metainterfaces with specified friction laws, *Science* 383:200-204, 2024.

[2] A.I. Vakis, et al., Modeling and Simulation in Tribology Across Scales: an Overview, *Tribol. Int.* 125:169-199, 2018.

Thursday
23 April 2026

Recent Advances in Third Medium Contact

P. Wriggers, Bing-Bing Xu, J. Korelc

Leibniz University Hannover, Germany

The third medium contact approach has been successfully employed in structural applications and extended to various optimization problems. This discretization technique replaces classical contact formulations and algorithms by introducing a compliant interfacial layer - referred to as the third medium - between the contacting bodies. Unlike traditional contact methods, this formulation naturally accommodates finite deformations at the interface. As the two bodies approach each other, the third medium undergoes compression and effectively acts as a deformable barrier, preventing interpenetration and transmitting contact forces in a smooth and numerically stable manner.

The approach is also applied to coupled problems, with focus on thermo-mechanical analysis. Then heat conduction must be incorporated into the model, which typically requires specialized interface laws when using classical contact formulations. These laws aim to capture the complex thermal behavior at the contact interface, including discontinuities and varying conductance. In contrast, the third medium approach offers the advantage to account for the interface behavior without the need for additional interface conditions. This includes the gradual heat transfer through the surrounding gas when the bodies are near each other, as well as the localized heat conduction that occurs upon physical contact. As a result, the third medium naturally captures both non-contact and contact-phase thermal conduction within a unified framework.

In this talk, we discuss the different approaches that can be applied within third media contact discretization schemes for linear and quadratic ansatz functions. The discretization is carried out using finite and virtual elements and then applied to mechanical and thermo-mechanical contact problems in two- and three-dimensions. The presentation includes comparison of different discretization schemes and algorithms in the light of robustness, efficiency and accuracy

Analytical Modeling and Experimental Validation of Ball–Beam Impact Dynamics Using a Nonsmooth Contact Dynamics Framework

Katarina Tutić, Teo Mudrić and Nina Čeh

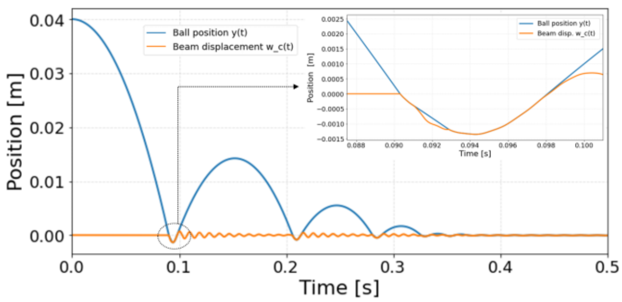
Faculty of Civil Engineering, University of Rijeka

Impacts between rigid bodies and flexible structures arise in many engineering applications and can exhibit complex contact phenomena that challenge current modeling approaches. This work considers a benchmark configuration of a rigid ball falling on a simply supported Euler–Bernoulli beam. The objective is to formulate and validate an integrated set of analytical, experimental, and numerical results for this benchmark configuration, with particular emphasis on identifying (i) when and why sub-impacts occur and (ii) under which conditions the simplified model (rigid ball – elastic beam) is applicable.

An analytical model is developed using a piecewise approach that separates the motion into distinct phases. During the free-flight phase, the ball moves independently while the beam response is governed by modal analysis. The contact phase is characterized by coupled dynamics in which the ball is modeled as a point mass at the contact location, enabling unified ball-beam motion [1]. Contact constraints govern transitions between phases: impact begins when the bodies touch, while separation occurs when contact is lost.

Experimental validation is achieved through a carefully designed setup that ensures repeatable initial conditions. A steel ball is released from controlled heights using an electromagnet (Figure 1a). Structural response is captured using a stereo Digital Image Correlation system (ZEISS ARAMIS 4M), enabling detailed kinematic reconstruction and precise measurement of displacement, velocity, and acceleration histories.

(a)



(b)

Figure 1: Ball and beam model: (a) experimental setup, (b) ball and midspan beam displacement over time, highlighting sub-impacts.

In parallel, numerical solutions of the governing equations are obtained using a finite-element beam model combined with a nonsmooth contact dynamics (NSCD) Moreau–Jean time-stepping scheme (Figure 1b) [2]. The numerical approach can be extended to a wide range of significantly more complex configurations, which motivates its investigation in this work. Through systematic comparison of all three methodological approaches on this benchmark problem, the work clarifies the capabilities and limitations of analytical models and current NSCD numerical schemes for rigid–flexible impact dynamics.

References

[1] Goldsmith, W. (1960). Impact: The Theory and Physical Behaviour of Colliding Solids. Edward Arnold Publishers Ltd., London.

[2] Jean, M. (1999). "The Non-Smooth Contact Dynamics Method." Computer Methods in Applied Mechanics and Engineering, 177(3–4), 235–257.

Evolution of the contact between rough viscoelastic solids after decreasing loads: memory erasure and monotonic increase

Zichen Li, Renald Brenner and Lucas Frérot

Sorbonne Université, CNRS, Institut Jean Le Rond d'Alembert, F-75005 Paris, France

The true contact area partly determines the friction force, yet its evolution in rough viscoelastic interfaces under decreasing loads remains incompletely understood. In experiments where a rough contact between polymethyl-methacrylate blocks is unloaded, Dillavou and Rubinstein [1] observed that, after the load is reduced, the contact area decreases over long times, characteristic of long-term memory in glassy systems. However, the modeling elements essential to reproducing the area decrease and long-term memory remain uncertain. Here, we investigate these effects with linear fractional viscoelastic rough-contact models [2]. Extending established contact theories and numerical schemes to fractional viscoelasticity (characterized by a broad relaxation spectrum), we reproduce the logarithmic ageing widely observed under steady load, but find that unloading erases any memory of the previous contact area. The contact reacts as if it had always been subjected to the reduced load, even over short times, unlike the behavior of a standard linear solid. Furthermore, none of our simulations display a post-unload decrease in contact area; we demonstrate analytically that no linear viscoelastic model is able to reproduce the area decrease. Consequently, local internal variables must be introduced to account simultaneously for persistent contact memory and the observed reduction of the contact area after unloading.

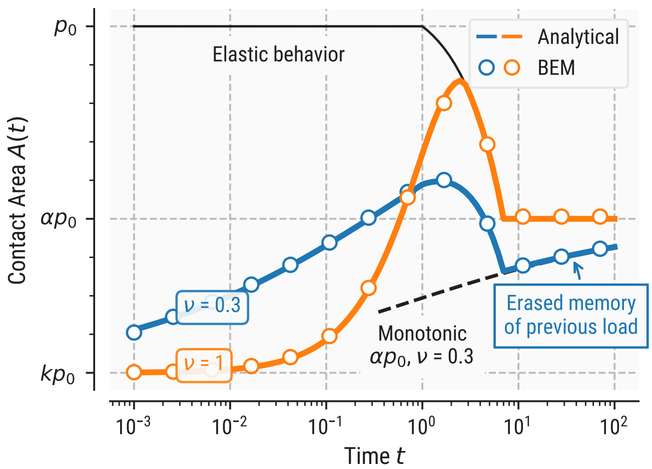


Figure 1: Evolution of the true contact area under constant load p_0 until time $T = 1$, then linear load decrease until $T + \Delta T = 6$, with load held constant at αp_0 afterwards. The exponent ν controls the breadth of the relaxation spectrum ($\nu = 1$ is a standard linear solid, $\nu = 0.2$ has a wide spectrum).

References

[1] Sam Dillavou and Shmuel M. Rubinstein. Nonmonotonic Aging and Memory in a Frictional Interface. *Physical Review Letters*, 120(22):224101, June 2018.

[2] Zichen Li, Renald Brenner, and Lucas Frérot. Evolution of the contact between rough viscoelastic solids after decreasing loads: Memory erasure and monotonic increase. (arXiv:2511.19402), November 2025.

Effect of velocity ratio on slip field during oblique landing of elastomer spheres

A. Mille, D. Dalmas and J. Scheibert

CNRS, Ecole Centrale de Lyon, ENTPE, Laboratoire de Tribologie et Dynamique des Systèmes, UMR5513, 69134 Ecully, France

Under increasing shear, the contact between a rough elastomer and a rigid surface, for example a tire on a road surface, may undergo significant changes in terms of morphology of the micro-contacts due to the presence of wear [1]. In this context, our research is motivated by the observation of surfaces characterized by heterogeneous wear patterns [2] that may originate from heterogeneous stress fields within the contact interface.

To better understand such contact heterogeneities, in this study, we analyze the displacement fields of an elastomer sphere subjected to kinematics that are representative of the local behavior during rolling. It consists in an "oblique landing", where normal and shear displacements increase simultaneously. These analyses are performed by measuring in-operando not only the classical evolution of macroscopic normal and tangential forces but also that of the true contact area and interfacial displacement fields.

To do so, we conduct experiments on single normal loading/unloading cycles during a continuous shearing motion, simulating local contact stimuli during one tire revolution. We use an opto-mechanical device recently developed in our laboratory [3]. The latter enables complex contact loading with five simultaneous and independent degrees of freedom and high-resolution monitoring of all three forces and three moments at the contact interface. It also enables high-resolution visualization of the contact area, enabling in-operando measurements within the real contact area of the tangential displacement field through advanced image analysis techniques [4].

This experimental procedure allowed us to study the influence of key parameters, such as the ratio between "landing" and sliding speed or the maximal normal force. First, we performed preliminary tests on unworn spheres of uncharged elastomer (PDMS) seeded with markers to track the evolution of contact area and interfacial displacement fields via image analysis. Then, we compare our experimental measurements with predictions from a new analytical model, developed by extending existing frameworks from the literature [5]. It enabled us to identify and predict which contact state prevails at any instant of the cycle (sticking, partial slip or full sliding) as a function of the velocity ratio. Subsequent tests on pristine tire-type rubber extended our understanding to more realistic rubber materials and allow us to link wear patterns, interfacial fields and variation of macroscopic energy losses.

References

[1] G. Petitet, M. Barquins, Matériaux caoutchouteux - Morphologie, formulation, adhérence, glissance et usure, ed. p.p.e.u. romandes, 2008.

[2] M. Huang, et al., P., A new test method to simulate low-severity wear conditions experienced by rubber tire materials, Wear 410-411, 72-82 (2018)

[3] M. Guibert, C. Oliver, T. Durand, T. Le Mogne, A. Le Bot, D. Dalmas, J. Scheibert, and J. Fontaine, A versatile flexure-based six-axis force/torque sensor and its application to tribology, Review of Scientific Instruments 92, 085002 (2021)

[4] J. Lengiewicz, …, C. Courbon, D. Dalmas, J. Scheibert, J. Mech. Phys. Solids 143, 104056 (2020)

[5] K. L. Johnson, Contact Mechanics, Cambridge University Press, 1985

Semi-analytical model of periodic coated viscoelastic contact.

N. Durand[1,2], A. Platzer[1], A. Duval[1], T. Chaise[1], D. Nelias[1]

[1] *INSA Lyon, CNRS, LaMCoS, UMR5259, 69621 Villeurbanne, France.*

[2] *Manufacture Française de Pneumatiques Michelin, 63040 Cedex 9 Clermont-Ferrand.*

Semi-analytical contact models (SAMs) are faster to compute than finite element, particularly when modelling viscoelastic contact [1]. However, these methods generally only offer linear viscoelastic models. This paper proposes a layered periodic formulation [2, 3] in which each layer exhibits distinct viscoelastic response, thus increasing the complexity of the material's behaviour.

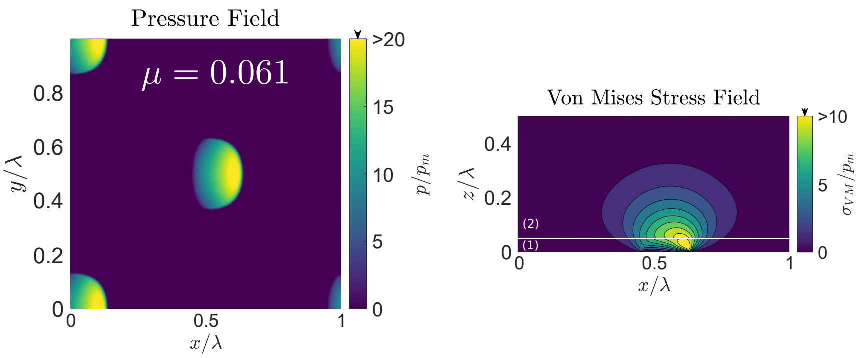


Figure 1: Layered diagram

The left side of Figure 1 shows the pressure field of the contact between a flat coated vicoelastic surface and a periodic bi-sine rigid surface. This field exhibits asymmetry between the front and rear of the contact, with the right side of the contact being more heavily loaded than the left side. This is due to dissipation causing a delay in the mechanical resonse of the body, between the front and rear of the contact.

Furthermore, the right-hand side of Figure 1 shows a cross-section of the equivalent von Mises stress in the xz plane. Between layers (1) and (2), a clear discontinuity is visible in the largest yellow area. This yellow area is larger and more widespread in layer (2) than in layer (1). This is a consequence of a well-known result in contact mechanics: the maximum von Mises stress is always found in the sublayer rather than at the contact point, provided there is no Coulomb friction. It is also due to the material in layer (2) having greater viscoelastic dissipation than that in layer (1), which explains the greater asymmetry in layer (2).

References

[1] K. E. Koumi, D. Nelias, T. Chaise, and A. Duval. Modeling of the contact between a rigid indenter and a heterogeneous viscoelastic material. *Mechanics of Materials*, 77:28–42, 2014.

[2] E. R. Wallace, T. Chaise, and D. Nelias. Rolling contact on a viscoelastic multi-layered half-space. *International Journal of Solids and Structures*, 239-240:111388, 2022.

[3] N. Durand, A. Platzer, A. Duval, D. Nélias, and T. Chaise. Coupled effect of viscoelasticity and roughness scales on hysteretic friction in sliding periodic contact. *Journal of the Mechanics and Physics of Solids*, page 106261, 2025.

A frictional contact formulation for planar mechanisms

P.D. Maddio, A. Cammarata and R. Sinatra

Department of Civil Engineering and Architecture,
University of Catania,
Via S. Sofia 64, 95123, Catania, Italy
pietro.maddio@unict.it

Frictional contact plays a crucial role in the dynamic behavior of mechanical systems, influencing energy dissipation, motion stability, and force transmission [1, 2]. Accurate modeling of tangential interactions remains a challenge, particularly when combined with flexible body dynamics. This work extends a recently proposed area-based contact formulation [3] to include frictional effects. The original method relies on the penalty approach to compute the normal contact force between two planar bodies by exploiting the geometric overlap: the force magnitude is proportional to the intersection area through a penalty coefficient, its direction is obtained from weighted projection vectors, and it is applied at the centroid of the overlapping region. For flexible bodies, this global force is decomposed into nodal contributions that satisfy both force and moment equilibrium. The present contribution introduces tangential forces through a regularized Coulomb friction model. The relative sliding velocity at the contact centroid is evaluated considering both translational and rotational motion of the contacting bodies. A sigmoid function ensures a smooth transition between sticking and sliding regimes, avoiding numerical instabilities at near-zero velocities. The resulting friction force, proportional to the normal force magnitude, is then distributed among the contact nodes while preserving static equivalence with respect to the global tangential force. As shown in Fig. 1, both the normal and tangential global forces are decomposed into nodal contributions acting on the contact interface. The extended formulation has been validated on planar mechanisms involving flexible bodies, demonstrating stable behavior and accurate force transmission under dynamic frictional contact conditions.

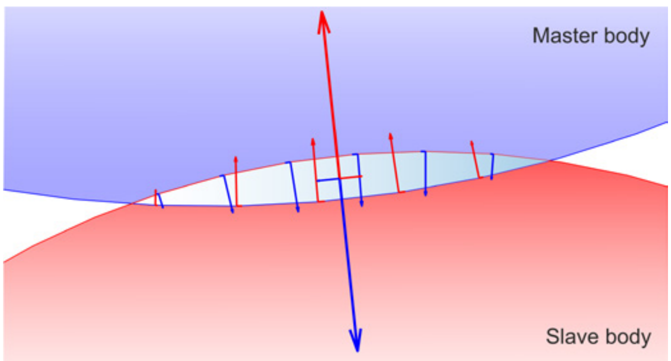


Figure 1: Normal and tangential global forces at the contact centroid with the corresponding nodal decomposition.

References

[1] P. Wriggers. *Computational Contact Mechanics*, 2nd ed., Springer, 2006.

[2] T. Laursen and J.C. Simo, A continuum-based finite element formulation for the implicit solution of multibody, large deformation frictional contact problems, *Int. J. Num. Meth. Engng.*, 36:3451–3485, 1993.

[3] P.D. Maddio, R. Sinatra and A. Cammarata, A novel 2D penalty formulation for frictionless contact, *Multibody System Dynamics*, 1-25, 2025.

Partial Slip in Contact of an Elastic Quarter-Space

Qiang Li

Institute of Mechanics, Technische Universität Berlin, 10623, Germany

In contact mechanics, many classical elastic contact theories, such as Hertzian theory, are based on half-space assumptions and therefore do not account for boundary-induced edge effects. However, in many engineering applications—such as gears, bearings, and indentations near specimen edges—contact frequently occurs in the vicinity of free surfaces, where stress and deformation fields are strongly influenced by nearby boundaries. The elastic quarter-space, a semi-infinite body bounded by two orthogonal free surfaces, provides a fundamental model for analyzing such edge effects. While recent studies have addressed normal contact on elastic quarter-spaces, both with and without adhesion [1–3], tangential contact remains relatively unexplored.

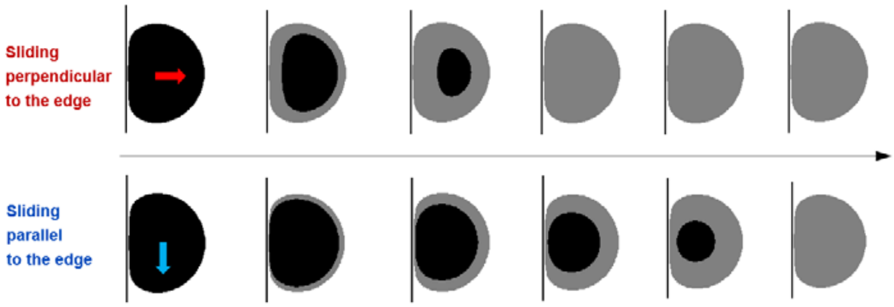


Figure 1: Evolution of stick (black) and slip (gray) region during sliding. Above: Sliding perpendicular to the edge; below: sliding parallel to the edge.

In this work, we investigate the partial-slip behavior of a rigid parabolic indenter in contact with an elastic quarter-space. Hetényi's method of overlapping two elastic half-spaces is extended to incorporate tangential loading applied to both the top and side surfaces. Equivalent stress distributions on the half-spaces are derived and used to evaluate contact tractions and subsurface stress fields. The method is then applied to analyze partial slip under Coulomb friction, with particular emphasis on the influence of edge proximity on stick–slip transitions.

In this presentation, we focus on a simplified contact configuration in which the side surface is not completely free, but its displacement in the direction perpendicular to the edge is constrained. Under this boundary condition, the contact solution can be obtained in a straightforward manner. Two sliding directions are considered, as illustrated in Figure 1, to examine the effect of indenter location. It is found that the maximum tangential displacement until the gross sliding is nearly independent of the indenter position when sliding occurs parallel to the edge. In contrast, when sliding is perpendicular to the edge, the critical displacement is reduced and becomes strongly location-dependent. The limitations of the proposed model will also be discussed.

References

[1] Q. Li, P. Edge effect and indentation depth-dependent contact behavior in contact of an elastic quarter-space, *Int. J. Solids Struct.*, 285: 112552, 2023.

[2] Q. Li, V.L. Popov, Non-adhesive and adhesive contacts of an elastic quarter-or eighth-space with freely sliding sides, *Friction*, 12: 2052–2063, 2024.

[3] Q. Li, S. Jiang, V.L. Popov, Edge Effects in Adhesive and Non-Adhesive Indentation: Experimental and Numerical Insights, *Eur. J. Mech. A/Solids*., 116: 105891, 2025.

A Point-Wise Complementarity Model for Frictional Contact Problems

Qingfeng Lou and József Kövecses

Department of Mechanical Engineering and Centre for Intelligent Machines, McGill University, Montréal, Canada

Contact problems in dynamic simulation remain challenging due to three intrinsic characteristics of the Coulomb friction law: coupling between normal and tangential forces; complementarity between velocities and forces; and the nonlinearity arising from friction forces that may point in any tangential direction while remaining bounded in magnitude.

The second order cone complementarity problem (SOCCP) model provides a mathematically consistent representation of ideal friction cone. However, it introduces a nonlinear augmented velocity term to construct the dual cone pairing with forces. Moreover, the three physical motion phases of a contact point: free (open), stick, and slide, are represented collectively within a single vector-wise orthogonality condition rather than being distinguished by separate inequality relations. These model level limitations propagate to associated fixed point and Newton type solvers, making convergence and stability difficult to ensure in practical computation.

The linear complementarity problem (LCP) model applies certain simplifications relative to SOCCP. A contact point is decomposed into one normal and two orthogonal tangential directions, and the friction bounds along these directions are predefined as fixed values. Thus each scalar component is treated independently and constrained to be nonnegative, reducing the vector-wise orthogonality to element-wise complementarity. As a result, the model corresponds to a friction cuboid, which neglects the intrinsic coupling and nonlinearity of Coulomb friction. These model level limitations propagate to associated LCP solvers such as Lemke, Murty, Judice, and classical projected Gauss–Seidel.

We introduce a model that provides a more physically coherent representation of frictional contact. It contains no augmented velocity term as in SOCCP and instead employs the constraint force and velocity pair directly. The model retains the decomposition into one normal and two tangential directions as in LCP, but the tangential bounds are defined as functions of the normal force and scaled by the tangential force or velocity directions. The force and velocity pair does not form a dual cone with vector-wise orthogonality and also breaks the element-wise complementarity structure. The model therefore lies outside both SOCCP and LCP, representing a contact point-wise complementarity with a direct physical interpretation. Its behavior is equivalent to an adaptive friction pyramid that becomes identical to the ideal friction cone at the computed solution, capturing the coupling, complementarity, and nonlinearity inherent in the Coulomb friction law.

The model can be solved through a two stage procedure: first solving the contact problem with a classical LCP solver such as projected Gauss–Seidel, then upon convergence classifying the motion phases and applying the rescaling needed to obtain the adaptive pyramid consistent with the ideal cone. The approach is self-contained, requiring no predefined friction bounds or external corrections from the dynamics model. It preserves the stability of standard LCP solvers while eliminating the coupling and nonlinear errors. We further implemented the approach within Vortex Studio, integrating it with its matrix data structures and achieving improvements in both physical accuracy and computational performance.

Finite Element Methods for Elastic Contact: Augmented Lagrangian and Nitsche

Tom Gustafsson[1] and Rolf Stenberg[2]

[1] *Department of Mechanical Engineering, Aalto University, Finland.* tom.gustafsson@aalto.fi

[2] *Department of Mathematics and Systems Analysis, Aalto University, Finland.* rolf.stenberg@aalto.fi

We consider three alternatives for solving elastic contact problems. The first one is a stable mixed method. For the Lagrange multiplier we either use the trace space of the field variable or a discontinuous piecewise polynomial. The first choice yields a stable method, cf., e.g., [1]. In the second one, the field value has to be augmented with bubble degrees of freedom to obtain stability [2]. For the latter choice the Augmented Lagrangian method has been proved to be very efficient to solve the linear system that arise. That applies also for the first choice, but due to the continuous Lagrange multiplier, the band-with of the penalty term is greater than that of the stiffness matrix from the field variable. In [3] it is shown that a bigger penalty term yields a faster convergence. There is however a trade-off with the conditioning of the system to be solved at each iteration. The requirement that this conditioning should be that of second order elliptic equations, leads to the choice $O(h^{-1})$ for the penalty parameter.

The second choice is to choose the mixed method with continuous Lagrange multiplier but based on the Augmented Lagrangian formulation of the continuous system, not the discrete. This choice appears to be in common use, but we have not found an analysis. By adapting the arguments in [3] we prove the convergence. There is, however, a fundamental difference between this and the first choice, if the penalty parameter is chosen too large, the convergence rate of the discretization is reduced. The analysis shows that the optimal choice is again $O(h^{-1})$.

This leads us to the third method, that of J.A. Nitsche. With the choice $O(h^{-1})$, the second choice resembles an unsymmetric Nitsche method. We introduce Nitsche in this spirit by adding a symmetrizing term. The final method is symmetric, optimally conditioned, and solved in one shot, no iteration needed.

References

[1] T. Gustafsson, P. Råback, J. Videman, Juha Mortaring for linear elasticity using mixed and stabilized finite elements. *Comput. Methods Appl. Mech. Engrg. 404* (2023)

[2] F. Ben Belgacem, The mortar finite element method with Lagrange multipliers. *Numer. Math.* 84 (1999) 173–197

[3] R. Glowinski, O. Pironneau, On numerical methods for the Stokes problem. Energy methods in finite element analysis, pp. 243–264. *Wiley Ser. Numer. Methods Engrg.* Wiley-Intersci. Publ. John Wiley & Sons, Ltd., Chichester, 1979, large deformation frictional contact problems, *Int. J. Num. Meth. Engng.*, 36:3451–3485, 1993.

A monolithically coupled chemo-mechano-thermodynamical contact formulation

Roger A. Sauer[1,2,3]

[1] *Institute for Structural Mechanics, Ruhr University Bochum, 44801 Bochum, Germany, E-mail: roger.sauer@rub.de*
[2] *Dept. of Structural Mechanics, Gdańsk University of Technology, 80-233 Gdańsk, Poland*
[3] *Dept of Mechanical Engineering, Indian Institute of Technology Guwahati, Assam 781039, India*

This work presents a general contact formulation for coupled chemical, mechanical and thermal contact. It is derived from the balance laws of mass, momentum and energy at the contact interface, thus exposing the general coupling between the different fields [1]. Chemical bonding and debonding is assumed to occur between bonding sites following a kinetic reaction equation. A quadratic contact potential is used to describe the free energy of contact, bonding and debonding. The proposed formulation corresponds to the known thermo-mechanical contact equations [2] extended to chemical contact, see Figure 1. The formulation is discretized and implemented in the 3D nonlinear finite

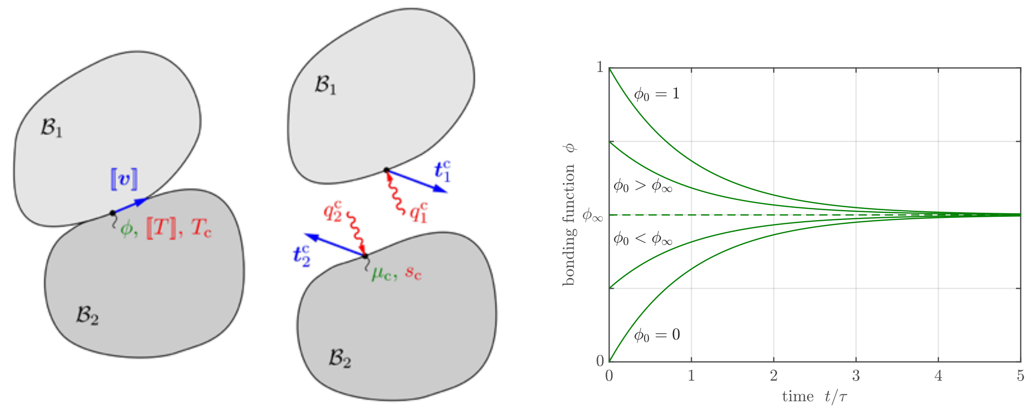


Figure 1: Thermo-chemo-mechanical contact [1]: (a) Bodies in contact with their chemical, mechanical and thermal contact fields. (b) Exemplary evolution of the contact bonding state for various initial conditions.

element framework of [3] using a fully monolithic coupling formulation. Implicit time integration is used of the chemical evolution equations. The formulation is illustrated by several applications. A particular focus is placed on implant osseointegration [4] and thermo-chemo-mechanical bonding.

References

[1] R.A. Sauer, T.X. Duong and K.K. Mandadapu, A chemo-mechano-thermodynamical contact theory for adhesion, friction and (de)bonding reactions, *Math. Meth. Solids.*, 27(4):711-745, 2022.

[2] T.A. Laursen, *Computational Contact and Impact Mechanics: Fundamentals of modeling interfacial phenomena in nonlinear finite element analysis*, Springer, Berlin, 2002.

[3] R.A. Sauer, and L. De Lorenzis, An unbiased computational contact formulation for 3D friction, *Int. J. Numer. Meth. Engng.*, 101(4):251-280, 2015.

[4] K. Immel, V.-H. Nguyen, G. Haïat and R.A. Sauer, Modeling the debonding process of osseointegrated implants due to coupled adhesion and friction, *Biomech. Model. Mechanobiol.*, 22:133-158, 2023.

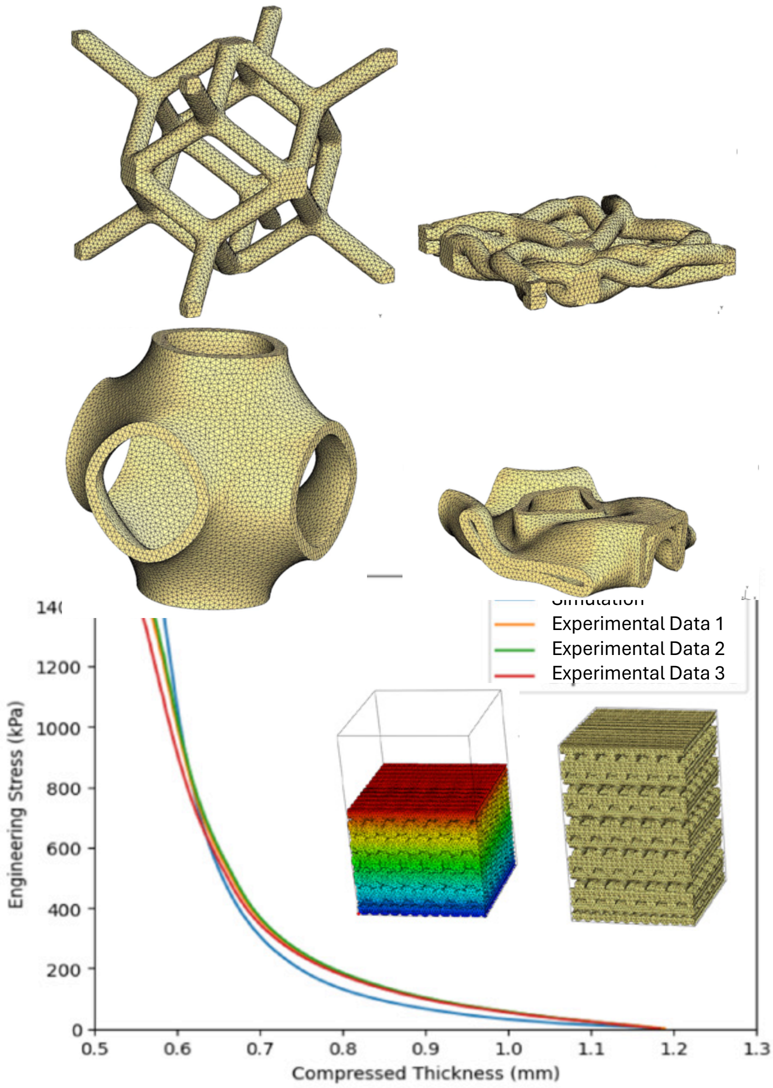
Where is contact now?
(a code developer's perspective)

Of course that is rhetorical question, its everywhere and typically a critical part of any complex mechanical model. But like many other areas of computational mechanics, it gets obscured by some of the current technical (and cultural) trends. In this talk, I will review some of the historical trends that influenced *me* as a code developer and how computational contact mechanics may fit in with the latest's trends of AI, GPU programming and Digital Twins. These trends seem unrelated to traditional areas such as contact mechanics, but I claim that these trends may offer new challenges for people working on contact methods in the areas of more robust/accurate solvers with the ability to handle larger and more complex analyses, computer science and more. In this talk, I will present some of the recent offerings from our lab which represent applications that could fit into this new technical world such as iterative linear solvers, optimization, geomechanics, energy consistent friction, architected materials and digital twins of direct ink written (DIW) polymers. Much of this work becomes multidisciplinary between mathematicians, CS folks and physicists who quickly come to appreciate the numerical complexity and challenges contact mechanics presents and the field of study.

(a)

(b)

(c)



Examples of unit cells for (a) the rhombic dodecahedron lattice (b) and TPMS (Triply Periodic Minimal Surface)–derived shell structure and (c) simulation of a uniaxial compression test compared to experimental results.

Extending Nitsche's method for the unilateral contact of beams

M. Schorsch[1,2], Y. Renard[2,3], D. Dureisseix[2], T. Elguedj[2], and C. Pozzolini[1]

[1] *Framatome, Paris/Lyon, F-92400 COURBEVOIE, cedric.pozzolini@framatome.com*
[2] *INSA Lyon, CNRS, LaMCoS, UMR5259, F-69621 Villeurbanne*
{matthieu.schorsch, yves.renard, david.dureisseix, thomas.elguedj}@insa-lyon.fr
[3] *INSA Lyon, UJM, UCBL, ECL, CNRS, Institut Camille Jordan, UMR5208, F-69621 Villeurbanne*

Non-linearities and non-smooth behaviors caused by impact phenomena give rise to complex numerical problems in the form of variational inequalities that require appropriate and effective numerical methods. Numerical methods for contact problems have been an active area of research for many years, but new methods continue to emerge. One of them is the Nitsche method. Initially introduced by Nitsche [1] to impose Dirichlet conditions in a 'weak' manner, it has since been applied to various interface problems. Applications to 3D contact mechanics have been presented and mathematical analyses have been published (see, for example, [2, 3]). Unlike all other methods, Nitsche's method is variationally consistent (and converges optimally) and does not introduce any additional degrees of freedom. Despite the advantages of this method, the realistic contact forces it delivers are partly due to the use of a 3D stress tensor. The question of extending Nitsche's method to thin 1D and 2D structures therefore arises.

In this work, we extend Nitsche's method to beam contact, with an approach similar to what was done by Fabre et. al. [4] for plate models. In particular, we show that the classic Euler Bernoulli and Timoshenko beam models have kinematics that are too 'poor' in the thickness of the structure to benefit from the theoretical advantages of Nitsche's method. Indeed, for these two models, the Nitsche contact formulation obtained is equivalent to that of a penalty contact. We therefore propose a beam model with enhanced kinematics, introducing a term that models pinching in the thickness of the beam, and show that this allows us to implement a relevant Nitsche contact formulation. The relevance of the method is assessed by comparing its results with the classical Timoshenko model, where contact is implemented using the penalty method.

[1] J. Nitsche. Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind. *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, 36(1):9–15, July 1971.

[2] Franz Chouly and Patrick Hild. A Nitsche-Based Method for Unilateral Contact Problems: Numerical Analysis. *SIAM Journal on Numerical Analysis*, 51(2):1295–1307, January 2013.

[3] Franz Chouly, Mathieu Fabre, Patrick Hild, Rabii Mlika, Jérôme Pousin, and Yves Renard. An Overview of Recent Results on Nitsche's Method for Contact Problems. In Stéphane P. A. Bordas, Erik Burman, Mats G. Larson, and Maxim A. Olshanskii, editors, *Geometrically Unfitted Finite Element Methods and Applications*, volume 121, pages 93–141. Springer International Publishing, Cham, 2017. Series Title: Lecture Notes in Computational Science and Engineering.

[4] Mathieu Fabre, Cédric Pozzolini, and Yves Renard. Nitsche-based models for the unilateral contact of plates. *ESAIM: Mathematical Modelling and Numerical Analysis*, 55:S941–S967, 2021.

A mortar-type computational approach for the simulation of wear in the presence of third bodies and transfer film

T. Silva Sabino[1,2], A. Couto Carneiro[2], F. Andrade Pires[1,2]

[1] *FEUP — Faculty of Engineering of the University of Porto, Porto, Portugal.*
[2] *INEGI — Institute of Science and Innovation in Mechanical and Industrial Engineering, Porto, Portugal.*
Emails: sabino@fe.up.pt, acarneiro@inegi.up.pt, fpires@fe.up.pt

The concept of a *third-body* in tribology refers to particles or layers that form between two contacting surfaces during sliding or rolling motion [1]. Third-bodies and transfer film layers can significantly alter wear behaviour, acting as solid lubricants, carrying load, and protecting the primary surfaces from damage. Despite their near-ubiquity in real-world tribological systems, third-body effects are often neglected in modelling. Accurately capturing third-body effects is crucial for predicting the lifespan and reliability of mechanical components. However, numerical simulation of their behaviour remains challenging due to the complex interactions between contact mechanics, material removal, and debris dynamics, including diffusion and accumulation, and the complex interaction with other tribological properties.

This work presents a numerical approach for simulating wear in the presence of third-bodies using the finite element method. A macroscale modelling strategy is proposed that incorporates the formation, evolution and loss of third-bodies within a tribological system (see Figure 1). The approach leverages a state-of-the-art dual-mortar contact formulation, enabling robust and accurate treatment of non-penetration and frictional constraints. By representing wear and third-body particles via a state variable field [2], the model effectively captures their impacts on contact interactions, wear rates, and frictional responses. A management strategy for the state variables is implemented to account for debris diffusion. The model is validated through numerical examples, demonstrating its ability to capture complex interactions in scenarios such as fretting wear and reciprocating sliding. The proposed approach provides a deeper understanding of a phenomenon often overlooked in wear simulations, and not only unlocks new predictive capabilities but also guides the design of more durable tribological systems.

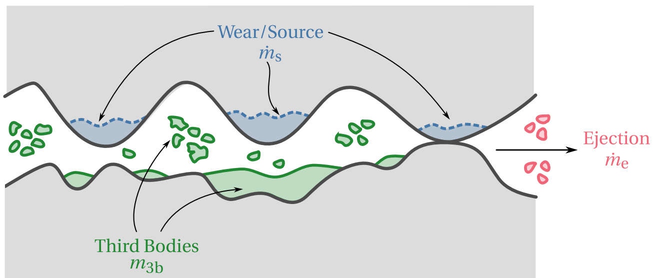


Figure 1: Schematic representation of a tribological system with third-body particles between two bodies in contact.

References

[1] M. Godet, The third-body approach: A mechanical view of wear, *Wear*, 100(1):437–452, 1984, DOI: 10.1016/0043-1648(84)90025-5.

[2] N. Strömberg, An augmented Lagrangian method for fretting problems, *European Journal of Mechanics-A/Solids*, 16(4):573–593, 1997, DOI: 10.1016/S0997-7538(97)80024-1.

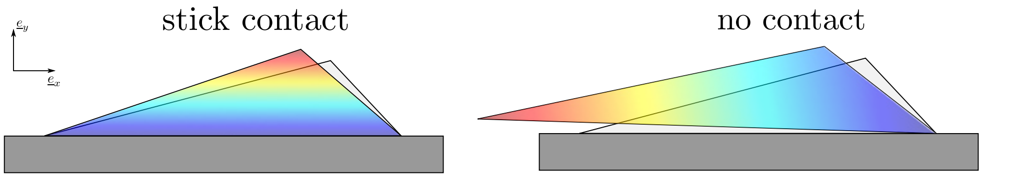
Comparison of three algorithms on frictional contact problem with two distinct solutions

Valentine Rey

Nantes Université, Ecole Centrale Nantes, CNRS, GeM, UMR 6183, F-44000 Nantes, France
valentine.rey@univ-nantes.fr

The Coulomb law of friction is widely used to model the contact between two solids. It is well-known that mathematical results on the existence and uniquement of solution for contact problems with Coulomb friction are limited to small friction coefficient [1]. This is due to the fact that this law cannot be derived from a potential but only from a bi-potential. As a consequence, some contact problems may have one, one or multiple (up to infinity) solutions.

In this paper, we consider the quasi-static frictional contact between one elastic body and one rigid obstacle. This two-dimensional academic example coming from [2] exhibits multiple solutions when the friction coefficient $\mu \geq 3$. Indeed, one solution with no contact and one solution with stick contact, illustrated in Figure 1, both verify the problem's equations.



$\underline{u}_{ex,1} = -6\alpha y \underline{e}_x - \frac{3}{4}\alpha y \underline{e}_y$ $\quad$ $\underline{u}_{ex,2} = (7x + y - 7)\alpha \underline{e}_x + \alpha(1 - x - \frac{7}{4}y)\underline{e}_y$

Figure 1: Two solutions for problem [2]

Many numerical methods are available to solve contact problems involving friction [3]. Here, we choose 3 methods: the augmented Lagragian technique with Uzawa algorithm [4], the Nitsche method [5] and the primal-dual method [6] (convex optimization approach with cone constraints). We aim at studying the influence of the parameters of each method (usually defined by the user) on the nature of the converged solution. We show that both the initialization fields and the values of penalty parameters or step length do have an influence on the nature of the obtained solution and that the no contact solution is more likely to be found. It highlights the crucial need of verification tools and guidelines dedicated to users for such numerical methods.

References

[1] C. Eck, J. Jarusek, and M. Krbec, *Unilateral contact problems : variational methods and existence theorems*, CRC Press, 2005.

[2] G. Hild, Multiple solutions of stick and separation type in the signorini model with Coulomb friction, *ZAMM-Journal of Applied Mathematics and Mechanics*, 85(9):673–680, 2005.

[3] P. Wriggers. *Computational Contact Mechanics*, 2nd ed., Springer, 2006.

[4] J.C. Simo and T. Laursen, An augmented Lagrangian treatment of contact problems involving friction, *Computers and Structures*, 42:97–116, 1992.

[5] Y. Renard. Generalized Newton's methods for the approximation and resolution of frictional contact problems in elasticity, *Computer Methods in Applied Mechanics and Engineering*, 256 :38–55, 2013.

[6] Y. Kanno, Primal-dual algorithm for quasi-static contact problem with Coulomb's friction *Journal of the Operations Research Society of Japan*, 65:1–22, 2022.

A generalised Comninou contact model for interface cracks in anisotropic bimaterials

María A. Herrera-Garrido and Vladislav Mantič

Grupo de Elasticidad y Resistencia de Materiales, Escuela Técnica Superior de Ingeniería, Universidad de Sevilla, Camino de los Descubrimientos, s/n, Sevilla, 41092, Spain.

In interface cracks between dissimilar materials, contact between the crack faces in a region close to the crack tip is very common, particularly under mixed-mode loading conditions. This has been observed in numerous analytical and numerical studies. This feature distinguishes interface cracks from cracks in homogeneous materials and emphasizes the importance of considering frictional effects in the local mechanical response. Consequently, a contact model is required to obtain physically consistent descriptions of the crack tip fields.

This work investigates the asymptotic elastic fields associated with a semi-infinite interface crack with frictional contact between the crack faces in linear elastic anisotropic bimaterials under generalised plane strain conditions. The analysis is formulated within the framework of the Comninou contact model of interface cracks [1,2], combining contact conditions with the classical Coulomb friction law. Asymptotic power-law solutions are derived in [3] using the Stroh formalism of anisotropic elasticity [4,5].

The crack tip asymptotic solution is governed by two coupled parameters: the singularity exponent λ and the frictional shear stress direction ω, which coincides with the direction of relative sliding in the contact zone. These parameters are obtained from a system of two nonlinear eigenequations expressed in closed form in terms of bimaterial elastic properties. In contrast to many existing formulations, the sliding direction is not assumed a priori but emerges naturally from the solution of the eigenproblem.

Particular attention is given to solutions with $\lambda \geq 1$, which are relevant for the asymptotic description of frictional contact near the transition from stick to slip in anisotropic bimaterials. According to established results [6], no singular (unbounded) stresses occur at this transition, so only solutions with $\lambda \geq 1$ are physically meaningful in this context. These asymptotic solutions can also be applied to partial slip contact problems, such as the interaction between an indenter or punch and a sample, layer, foundation, or half-plane.

The results provide a consistent asymptotic description of frictional contact effects at interface crack tips in anisotropic bimaterials. The proposed framework is relevant for the modelling of contact-dominated interface fracture and can also be applied to the asymptotic analysis of partial slip contact problems.

References

[1] M. Comninou, The interface crack, J. Appl. Mech., 44:631–636, 1977.

[2] M. Comninou, Interface crack with friction in the contact zone, J. Appl. Mech., 44:780–781, 1977.

[3] M.A. Herrera-Garrido and V. Mantič, Stress singularities in the generalised Comninou frictional contact model for interface cracks in anisotropic bimaterials, J. Mech. Phys. Solids, 203:106214, 2025.

[4] A.N. Stroh, Dislocations and cracks in anisotropic elasticity, Phil. Mag., 3:625–646, 1958.

[5] A.N. Stroh, Steady state problems in anisotropic elasticity, J. Math. Phys., 41:77–103, 1962.

[6] J. Dundurs and M. Comninou, Some consequences of the inequality conditions in contact and crack problems, J. Elasticity, 9:71–82, 1979.

Dynamics with impact of elastic solids : issues, classical schemes and recent advances

Yves Renard

INSA Lyon, Université de Lyon, ICJ, LAMCOS, France.

The aim of the presentation is to propose an overview of the main difficulties encountered when constructing discretizations of dynamics with elastic solid impacts, particularly using spatial finite element discretization (difficulty to have energy conservation and avoiding spurious oscillations in particular).

One of the main specific difficulties in the construction of time integration schemes for the dynamics with impact of deformable solids, compared to other strongly non-linear problems, is that the finite element semi-discretization is a so-called measure differential inclusion which is notoriously ill-posed with very low regularity solutions. It is interesting to note that the ill-posed character of the finite element semi-discretization is not present in the case of the approximation of the contact condition by a penalty method [1]. The penalty method is however not consistent in the strong sense and induces an additional approximation. A second possibility is the use of Nitsche's methods which combines the fact of being strongly consistent and the well-posed character of semi-discretization (see [2]). In particular, in [3] a fully explicit schemes based on Verlet's scheme have been introduced, analyzed and compared to other previously introduced schemes for impact dynamics.

The presentation will focus on classical time discretizations of elastic structure impact problems and on some recent advances (Nitsche's method [3], CD-Lagrange FEM [4], space-time discretisation [5])

References

[1] D. Doyen, A. Ern, S. Piperno. Time-integration schemes for the finite element dynamic Signorini problem. SIAM J. Sci. Comput., 33(1):223--249, 2011.

[2] F. Chouly, M. Fabre, P. Hild, R. Mlika, J. Pousin, Y. Renard. An overview of recent results on Nitsche's method for contact problems. Lect. Notes in Comput. Sci. and Engng, 121:93--141, 2018.

[3] F. Chouly Y. Renard. Explicit Verlet time-integration for a Nitsche-based approximation of elastodynamic contact problems. Adv. Model. and Simul. in Eng. Sci., 5:31, 2018.

[4] Di Stasio, J., Dureisseix, D., Georges, G., Gravouil, A., Homolle. An explicit time-integrator with singular mass for non-smooth dynamics. *Computational Mechanics*, *68*(1), 97-112, 2021.

[5] V.A. Kovtunenkoa, A. Petrov, Y. Renard. Space-time FEM solution of dynamic contact problem with discontinuous velocity for multiple impact of deformed bar using PDAS method. Preprint.

A fractional model of finite viscoelasto-plasticity including damage through the Phase Field technique

Giacomo Petraglia[1], Ariel Ramírez-Torres[2], Alberto Sapora[1], Pietro Cornetti[1], Alfio Grillo[3]

[1] *Department of Structural, Geotechnical and Building Engineering, Politecnico di Torino, Italy*
[2] *School of Mathematics and Statistics, University of Glasgow, UK*
[3] *Department of Mathematical Sciences, Politecnico di Torino, Italy*

This work presents a monophasic modelling framework designed to describe the behavior of viscoelastic materials subjected to damage and plasticity in the finite deformation regime.

The model implements a multiplicative decomposition of the material's deformation gradient tensor into a viscoelastic, plastic and damage part [1]. The tensors associated with the viscoelastic and plastic distortions are computed, rather than being prescribed *a priori*, alongside the requirement that the related processes must be isochoric [2]. Damage distortions are prescribed from the outset assuming an isotropic behavior [1] and coupled to the Phase Field, intended as the *damage variable*.

The Phase Field is included into the constitutive framework via a grade-one model, thereby ensuring that the considered constitutive laws depend on the gradient of the Phase Field. Moreover, the model addresses viscoelasticity by assuming that the relation between mechanical stress and its associated strain rate be fractional in time [3].

The governing boundary-value problem and its computational background are outlined, followed by the presentation of relevant numerical results.

References

[1] M. Mićunović, *Thermomechanics of viscoplasticity*, Springer, 2009.

[2] A. Giammarini, A. Pastore, A. Ramírez-Torres, A. Grillo, A first-gradient approach to the remodeling and fluid flow in saturated porous media, *Math. Mech. Solids*, 0:1–39, 2025.

[3] T. C. da Costa-Haveroth, G. A. Haveroth, M. L. Bittencourt and J. L. Boldrini, A damage phase-field model for fractional viscoelastic materials in finite strain, *Comput. Mech.*, 69:1365–1393, 2022.

Friday
24 April 2026

Toward Real-Time, Stable, and Physically Accurate Simulation: Vertex Block Descent and Beyond

Anka He Chen

NVIDIA - Simulation Technology

Physics-based simulation demands three essential properties: stability, physical accuracy, and computational efficiency. Despite decades of progress, existing methods struggle to deliver all three simultaneously. Convergent solvers faithfully solve the underlying physics but suffer from instability and poor parallel performance due to global system dependencies. GPU-friendly approaches like Extended Position Based Dynamics (XPBD) achieve parallelism by simplifying the governing equations—at the cost of convergence to true physical behavior. Stability, meanwhile, remains elusive for all methods under strict computational budgets.

This presentation introduces Vertex Block Descent (VBD), a novel solver for the variational form of implicit Euler integration that achieves all three properties. By reformulating the problem as vertex-level Gauss-Seidel iterations that reduce global variational energy through purely local position updates, VBD enables massive parallelism while guaranteeing numerical convergence and unconditional stability.

Beyond the core solver, my work addresses the longstanding challenge of collision handling. I propose a formal universal collision formulation called Offset Geometric Contact (OGC), which guarantees intersection-free results and provides a universal collision formulation for codimensional objects—cloth, strands, and shells—achieving over 100× speedup compared to previous work. This is complemented by a novel penetration-free enforcement technique that is solver-agnostic and material-agnostic, enabling interactive simulation of extreme contact scenarios with millions of simultaneous contacts.

Finally, I will also present a high-resolution deformation capture system for acquiring real-world data on non-rigid objects, enabling data-driven simulation and opening new avenues for inverse physics problems.

Together, these contributions advance physics-based simulation by orders of magnitude in performance while providing unconditional stability and penetration-free guarantees under extreme conditions.

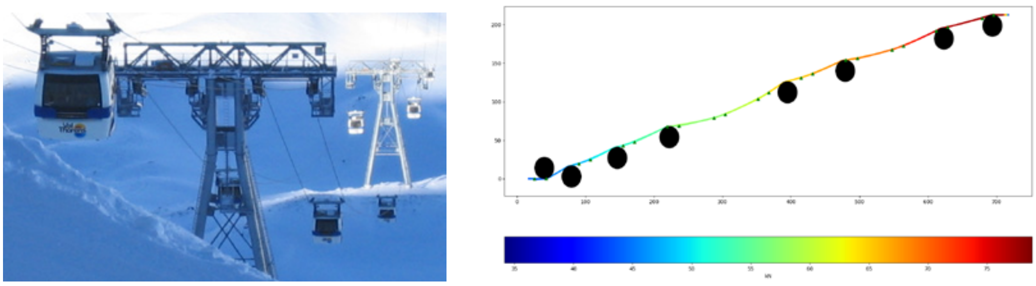
Dynamic cable simulation for ropeway transport systems

Charlélie Bertrand[1,2], Guillaume Mestdagh[1], Mathieu Weiss[3], Vincent Acary[1]

[1] *Tripop team, Inria, Grenoble, France*
[2] *ENTPE, Lyon, France*
[3] *Service technique des remontées mécaniques et des transports guidés, Grenoble, France*

Long confined to mountain ranges and ski resorts, cable transport systems are increasingly adopted in cities. Due to their fluid traffic and reduced ground footprint, urban cable transports offer an alternative solution to tramways and buses, as well as a new market for industrials. While the design of new installations usually relies on static analysis, past incidents involving dynamic effects, together with a trend for higher cable velocity, calls for a dynamical understanding of these systems. Although critical for safety, the dynamic simulation of cable transports is sparsely tackled in the literature, and no software exists to simulate an entire installation.

In this presentation, we propose a finite-element formulation for the dynamics of an elastic cable, subject to contact and friction with supports and sheaves. The cable motion equations are derived using the Lagrangian formalism, while Coulomb friction is handled using the framework of second-order cone complementarity. Our numerical program involves a custom finite element implementation in Python that avoids the appearance of compressive efforts in the cable, [1], and relies on solvers provided by the Siconos platform [2, 3] to solve for friction forces. We present dynamical simulation results based on a real chairlift installation, demonstrating the interest of our approach to anticipate unwanted dynamic effects. We then discuss the considered perspectives for this software, including the use of inextensible cable elements and interior-point optimization methods to manage contacts with supports.



(a) (b)

Figure 1: (a) Cable car installation in French Alps (source: Wikipedia), (b) Finite element simulation of an existing cable transport installation. The color bar represents the cable tension.

References

[1] C. Bertrand, V. Acary, C.-H. Lamarque, and A. Ture Savadkoohi. A robust and efficient numerical finite element method for cables. *International Journal for Numerical Methods in Engineering*, 121(18):4157–4186, 2020.

[2] Vincent Acary and Franck Pérignon. Siconos: A software platform for modeling, simulation, analysis and control of nonsmooth dynamical systems. *SNE Simulation News Europe*, 17(3/4):19–26, December 2007.

[3] Inria. Siconos, 2024. `https://siconos.org`.

Efficient mixed rod finite elements for rockfall protection ring-net barriers with frictional contact

J. Breuling

Institute for Nonlinear Mechanics, University of Stuttgart, Germany.

The protection of human infrastructure against natural hazards has become a major engineering challenge, particularly under the increasing impact of climate change. Among these hazards, rockfall poses severe risks to settlements, transportation routes, and critical infrastructure. Consequently, substantial effort is devoted to improving rockfall protection systems, including flexible ring-net barriers. To satisfy economic constraints and support optimal design, reliable numerical predictions of ring-net behavior have become indispensable, as illustrated in Figure 1.

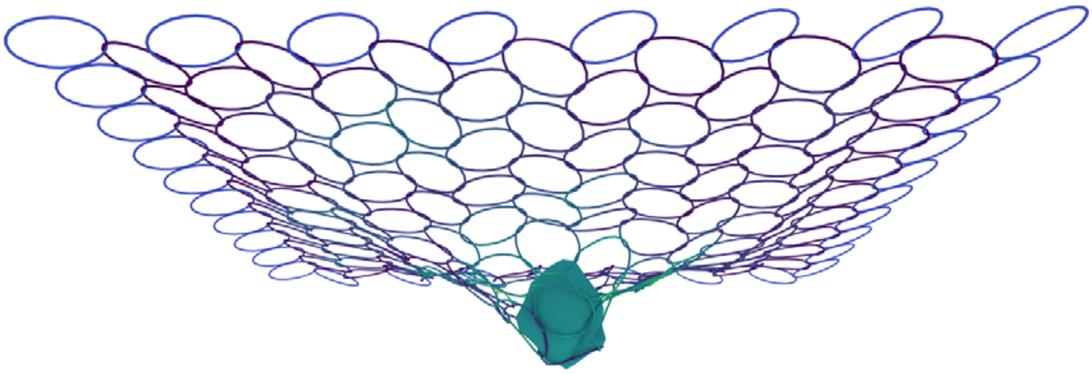


Figure 1: Numerical simulation of a rock impacting a rockfall protection ring net.

For the efficient numerical simulation of such highly deformable structures, we employ nonlinear rod finite elements as proposed in [1]. The formulation is based on a mixed finite element method [2] derived from the Hellinger–Reissner principle. A particularly efficient discretization is obtained using linear Lagrange interpolation with one-point quadrature for the internal forces, combined with a trapezoidal rule for dynamic and external contributions. To enhance performance further, the generally nonlinear internal force laws are differentiated in time, yielding an evolutionary system of equations. This allows the kinematics, dynamics, and internal forces to be integrated within a unified framework.

A complete description of the net-rock interaction requires an accurate treatment of frictional contact. For this purpose, the rod finite elements are embedded into the nonsmooth dynamics framework, resulting in a measure differential inclusion. The latter is discretized using a variant of Moreau's mid-point rule [3], adapted to the mixed rod finite element formulation with frictional contact and impacts.

Numerical experiments confirm the efficiency, robustness, and conceptual simplicity of the proposed method, demonstrating its suitability for large-scale simulations of ring-net rockfall protection systems.

References

[1] M. Herrmann, D. Castello, J. Breuling, I. C. Garcia, L. Greco, and S. R. Eugster. A mixed Petrov–Galerkin Cosserat rod finite element formulation, 2025.

[2] F. Brezzi and M. Fortin. *Mixed and Hybrid Finite Element Methods.* Springer New York, New York, NY, 1991.

[3] J. J. Moreau. Unilateral Contact and Dry Friction in Finite Freedom Dynamics. 1988.

Skin-pass Rolling as a Fluid-Structure-Contact Interaction Problem

Moritz Billen, Norbert Hosters and Marek Behr

RWTH Aachen University, Schinkelstraße 2, 52062 Aachen, Germany

Skin-pass rolling is the final step in producing aluminum sheets. During rolling, a lubricant is used to prevent the metal from sticking and to extend the lifetime of the roller. However, the lubricant also affects the surface of the sheet because some of it becomes trapped and takes part in the forming process.

From a modeling perspective, skin-pass rolling is a surface-coupled multi-physics problem. The interaction between the lubricant, the deforming aluminum, and the direct contact between the roller and the sheet cannot be ignored. This makes accurate fluid-structure-contact interaction models necessary.

In this work, we develop a partitioned approach in which the structural and fluid solvers are called one after the other. The roller is treated as rigid, the aluminum sheet is modeled with an elasto-plastic material law, and the fluid is described by the incompressible Navier-Stokes equations. To avoid instabilities caused by trapped fluid pockets, a Robin-Neumann coupling scheme is applied between the two solvers [1].

Contact changes the topology of the fluid domain, which is commonly handled through remeshing. Instead, we use the surface-reconstruction virtual region mesh update method, which is more cost-efficient. It adds a layer of inactive elements around the fluid domain that can be activated or deactivated based on boundary motion. Elements near the interface are updated to maintain a conforming boundary [2].

References

[1] T. Spenke, M. Make and N. Hosters. *Int. J. Num. Meth. Engng.*, 24(4), 979-997, 2023.

[2] F. A. González, S. Elgeti and M. Behr. The surface-reconstruction virtual-region mesh update method for problems with topology changes *Int. J. Num. Meth. Engng.*, 24(9), 2050-2067, 2023.

Variational formulation and numerical resolution of persistent adhesive contact in hyperelastic materials

Rawane Mansour, Mikaël Barboteu, Francesco Bonaldi and Serge Dumont

Laboratoire de Modélisation Pluridisciplinaire et Simulations, Université de Perpignan Via Domitia, 52 Avenue Paul Alduy, 66860 Perpignan, France.
rawane.mansour@univ-perp.fr, barboteu@univ-perp.fr, francesco.bonaldi@univ-perp.fr, serge.dumont@univ-perp.fr

This contribution develops an energy-consistent variational and computational framework for hyper-viscoelastic contact problems involving unilateral contact, adhesion, and friction under large deformations. The formulation extends the classical persistent contact condition to cohesive regimes, ensuring a consistent transition between full adhesion, progressive decohesion, and complete separation.

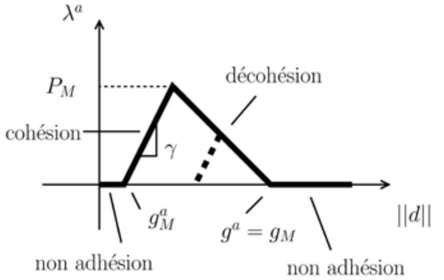


Figure 1: Normal adhesive traction λ_a versus adhesive gap $\|d\|$: cohesive (left), decohesion (middle), and non-adhesive (right) regimes.

Adhesion is described using a Talon–Curnier cohesive law, formulated as a projection-based relation driven by the accumulated interfacial elongation. The material behavior follows a Kelvin–Voigt hyper-viscoelastic model. The full formulation is expressed in the reference configuration, yielding a rigorous weak form and a straightforward finite element discretization. Let $\|d\|$ denote the adhesive gap, g^a the plastic interfacial elongation, and $s(g^a)$ the decreasing cohesive threshold; the normal adhesive traction is given by

$$\lambda^a = \mathrm{Proj}_{[0,\, s(g^a)]}\big(\gamma(\|d\| - g_0^a)\big), \tag{1}$$

where g_0^a denotes the previous value of the elongation at the beginning of the time step, and $\gamma > 0$ is a penalty parameter. Time integration is performed using an implicit midpoint scheme that preserves the discrete energy balance. The resulting nonsmooth problem is solved efficiently using a semi-smooth Newton method combined with a Primal–Dual Active Set (PDAS) strategy, eliminating Lagrange multipliers and ensuring robust convergence. Numerical experiments demonstrate the accuracy, stability, and energy consistency of the proposed approach in reproducing complex adhesion and decohesion mechanisms.

References

[1] C. Talon and A. Curnier. A model of adhesion coupled to contact and friction. *European Journal of Mechanics A/Solids*, 22:545–565, 2003.

[2] M. Hintermüller, K. Ito, and K. Kunisch. The primal-dual active set strategy as a semismooth Newton method. *SIAM Journal on Optimization*, 13(3):865–888, 2002.

Accurate contact detection for the predictive simulation of fibre assemblies

Emile Hohnadel, Octave Crespel, Thibaut Métivet and
Florence Bertails-Descoubes

Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LJK
e-mail: emile.hohnadel@inria.fr

The goal of this work is to improve simulation tools from the computer graphics community for the exploration of the mechanics of fibre assemblies. We focus on the curvature-based numerical model for Kirchhoff elastic rods, the so-called super-helix model [1] coupled with the non-smooth frictional contact solver so-bogus [2] to conduct the simulations. These numerical models have been validated geometrically [3] but lack validation in force. We have conducted a study on the three-point bending experiment, where we observe spurious jumps in the contact forces. We explain these jumps by the low-order (segment-based) detection method used in the simulations. We propose high-order contact detection methods in two and three dimensions that completely remove these issues [4]. We further illustrate the large-scale effect of a low-order contact detection method with a force study in hair combing, as illustrated in fig. 1.

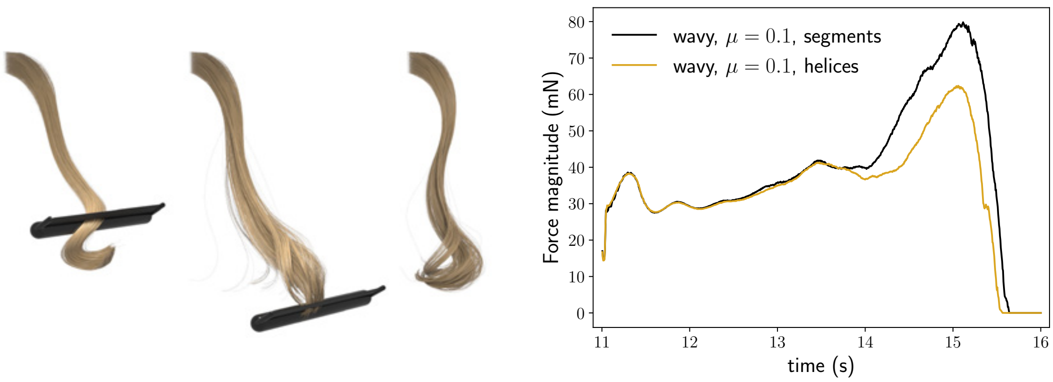


Figure 1: (left) Renders of a wisp of wavy hair during combing. (right) Comparison of the forces measured from the wisp on the comb between two contact detection methods, segment-based (in black) and exact (in yellow).

We present other protocols used to validate and calibrate the simulations against theoretical and experimental results. This lead to the study of the dissipative properties of a random architectured material [5]. Finally, we present a new two dimension implementation of the previous code more general and able to model more complex scenarios.

References

[1] F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy, J.-L. Lévêque. *Super-Helices for Predicting the Dynamics of Natural Hair*, ACM Trans. Graph. 25, 2006.

[2] G. Daviet, F. Bertails-Descoubes, L. Boissieux. *A hybrid iterative solver for robustly capturing coulomb friction in hair dynamics*, ACM Trans. Graph. 139, 2011.

[3] V. Romero, M. Ly, A.-H. Rasheed, R. Charrondière, A. Lazarus, S. Neukirch, F. Bertails-Descoubes. *Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact*, ACM Trans. Graph. 40, 2021.

[4] O. Crespel, E. Hohnadel, T. Metivet, F. Bertails-Descoubes. *Contact detection between fibres: high order makes a difference*, ACM Trans. on Graph. 43, 2024.

[5] T. G. Sano, E. Hohnadel, T. Kawata, T. Metivet, F. Bertails-Descoubes. *Randomly stacked open-cylindrical shells as a functional mechanical device*, Commun. Mater. 4, 2023.

A multiarea frictionless contact method for planar mechanism

A. Cammarata, P.D. Maddio and R. Sinatra

Dipartimento di Ingegneria Civile e Architettura,
Università di Catania,
Via S.Sofia 64, 95123, Catania, Italy
alessandro.cammarata@unict.it

Traditional discretizations of the contact interface have led to three primary schemes: node-to-node (NTN) [1], node-to-segment (NTS) [2], and segment-to-segment (STS) [3]. Referring to Fig. 1, the proposed method can be divided into three phases: a) – contact detection and calculation of the intersection area and its center of gravity, b)– calculation of the direction and magnitude of the global contact forces, c)– decomposition of the latter into nodal forces. The final contact force vector is thus fully defined by three elements: (i) its magnitude, proportional to the intersection area; (ii) its direction, given by the weighted average of the slave-to-master vectors; and (iii) its application point, located at the centroid of the intersection area. These components also allow the force to be decomposed into nodal contributions, enabling a detailed analysis of interactions in both rigid and flexible systems. Extensions to multiarea contact for non-conformal conjugate pairs has been considered. The method was tested on simple planar mechanisms and validation with Hertz theory and the STS method has been included.

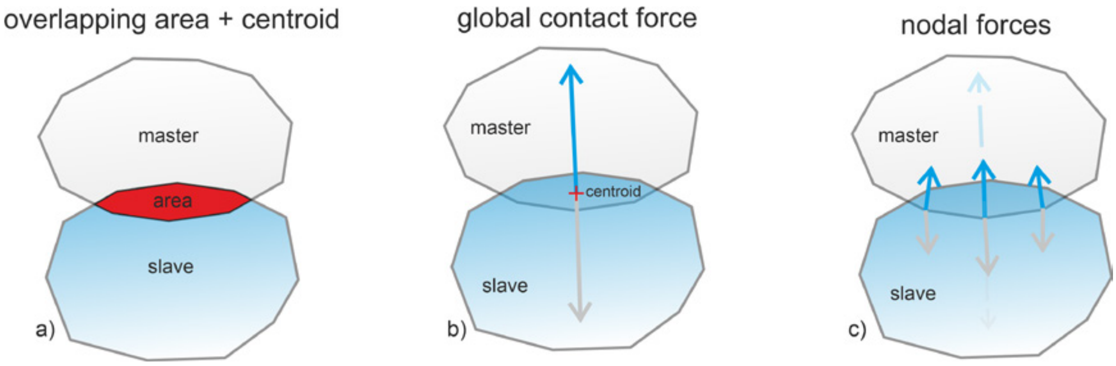


Figure 1: (a) computation of the overlapping area; (b) computation of the global force applied at the area centroid; (c) decomposition of the global force into an equilibrated system of nodal forces.

References

[1] A Francavilla and OC Zienkiewicz. A note on numerical computation of elastic contact problems. *International Journal for Numerical Methods in Engineering*, 9(4):913–924, 1975.

[2] Thomas JR Hughes, Robert L Taylor, Jerome L Sackman, Alain Curnier, and Worsak Kanoknukulchai. A finite element method for a class of contact-impact problems. *Computer methods in applied mechanics and engineering*, 8(3):249–276, 1976.

[3] Panayiotis Papadopoulos and Robert L Taylor. A mixed formulation for the finite element solution of contact problems. *Computer Methods in Applied Mechanics and Engineering*, 94(3):373–389, 1992.

Influence of numerical schemes on the apparent dynamics of the contact between a viscoelastic beam and two rigid stops

A. Khaddari[1,2], C. Pozzolini[2], C.Grenat[3], R.Alcorta[1], V. Lleras[4], Y. Renard[1]
[1] *INSA Lyon, CNRS, LaMCoS, UMR5259, 69621 Villeurbanne, France. ayman.khaddari@insa-lyon.fr, roberto.alcorta@insa-lyon.fr, yves.renard@insa-lyon.fr*
[2] *Framatome, 2 rue Professeur Jean Bernard, 69007 Lyon, cedric.pozzolini@framatome.com*
[3] *Framatome, Tour d'Areva, 1 place Jean Millier, 92400 Courbevoie, clement.grenat@framatome.com*
[4] *IMAG, 499-554 Rue du Truel 9, 34090 Montpellier, vanessa.lleras@umontpellier.fr*

In numerous applications of nuclear engineering and structural mechanics, many slender structures—notably steam generator tubes, fuel rods, and control rod drive mechanisms—can come into contact with rigid walls or abutments under dynamic loading. Accurate modeling and analysis of these phenomena are essential for evaluating the reliability, lifespan, and vibration resistance of these components under normal or accidental (earthquake) operating conditions.

Vibrating systems subjected to unilateral stresses exhibit inherently non-smooth dynamics characterized by the occurrence of impacts. The existence of chaotic regimes has been clearly established in several oscillator configurations subjected to abutments. However, for continuous beams, these phenomena are much less frequently described in the literature. Among the relevant contributions are the work of Liakou, Denoël, and Detournay [1] on the fast dynamics of beams with unilateral constraints, and the mathematical analysis by Mbengue [2], which provides existence and uniqueness results for a nonlinear beam model in contact with a foundation.

In industrial applications, contact is often modeled using a penalty method, allowing for low interpenetration and simplifying numerical processing. However, it is also possible to strictly impose non-interpenetration, leading to a contact force implicitly defined by the complementarity conditions. For the case of point obstacles, the work of Kuttler and Shillor [3] provided a detailed analysis of regularity and solutions. The present work is placed within a framework extended to the case of continuous and regular obstacles for a viscoelastic Euler-Bernoulli beam.

Semi-discretization using finite elements leads to differential inclusion, for which we exploit the singular mass method, initially developed by Renard [6] and adapted to thin structures by Pozzolini, Renard, and Salaün [7]. The beam is discretized in space using Hermite finite elements, necessary to handle a fourth-order spatial derivative operator. We then analyze the influence of the time scheme on the dynamics, in particular the effects related to scheme parameters, the penalty, and mass matrices. From a numerical point of view, several approaches coexist to enforce non-interpenetration: the penalty method, Lagrangian methods (pure or augmented) with complementarity conditions, or hybrid methods. Recall that convergence towards the reference solution is observed when the penalty coefficient remains within a range that guarantees both mechanical consistency and numerical stability. Beyond a certain threshold, the increased stiffness of the contact induces numerical oscillations and a loss of dynamic fidelity. This is one of the reasons that led us to investigate the emergence of non-repeatability during vibro-impact calculations. Therefore, we analyze here a simplified model of a beam subjected to repeated impacts, induced by an imposed sinusoidal displacement. Newmark schemes, whose stability in non-smooth contexts is still poorly documented in the literature, serve as the basis for the time integration [5].

We show that the observed dynamics—in particular the appearance or disappearance of bifurcations—depends crucially on the numerical choices. The objective of this study is therefore to provide a robust framework for the analysis of the non-smooth dynamics of beams in rigid contact, and to establish criteria for correctly interpreting the simulations.

References

[1] A. Liakou, V. Denoël, and E. Detournay. Fast In-Plane Dynamics of a Beam with Unilateral Constraints. *Journal of Engineering Mechanics*, 143(2):04016116, 2017.

[2] M. F. Mbengue. Analysis of a nonlinear beam in contact with a foundation. *ZAMM*, 65:783–796, 2014.

[3] L. Kuttler and M. Shillor. Vibrations of a beam between two stops. *Dynamics of Continuous, Discrete and Impulsive Systems B*, 8:93–110, 2001.

[4] Y. Dumont and L. Paoli. Vibrations of a beam between stops: convergence of a fully discretized approximation. *ESAIM: Mathematical Modelling and Numerical Analysis*, 40 (2006), 705-734

[5] Y. Dumont and L. Paoli. Vibrations of a beam between obstacles: convergence of a fully discretized approximation. *ESAIM: M2AN*, 40(4):705–734, 2006.

[6] Y. Renard. The singular dynamic method for constrained second-order hyperbolic equations. *J. Comput. Appl. Math.*, 234(3):906–923, 2010.

[7] C. Pozzolini, Y. Renard, and M. Salaün. The singular dynamic method for dynamic contact of thin elastic structures. *ESAIM: Proc.*, 42:20–33, 2013.

[8] C. Pozzolini and M. Salaun. Some energy-conservative schemes for vibro-impacts of a beam on rigid obstacles. *ESAIM: M2AN*, 45, 1163–1192, 2011.

Multilevel Augmented-Lagrangian Methods for Overconstrained Contact Discretizations

M. Weiser[1], F. Baumann[1] and R. Krause[2]

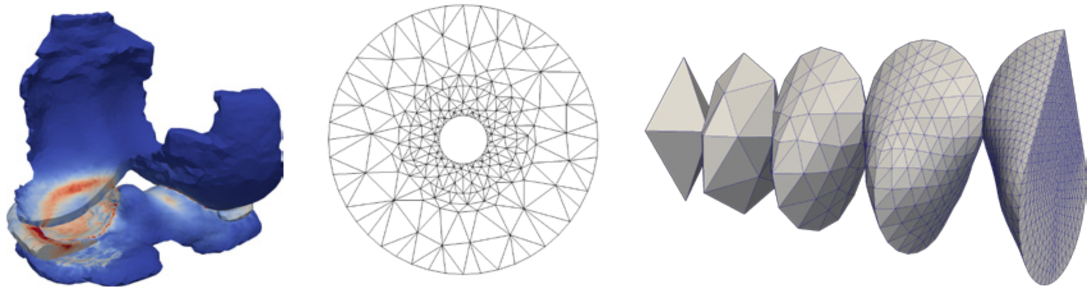
[1] {weiser,felix.baumann}@zib.de, Zuse Institute Berlin, Takustr 7, 14195 Berlin, Germany
[2] rolf.krause@kaust.edu.sa, King Abdullah U. Sci. Tech., Thuwal 23955-6900, Saudi-Arabia

The talk will address overconstrained formulations for multi-body contact and adapted multilevel solvers of augmented Lagrange type for the resulting QPs, with application to stress identification in articulate cartilage [2], see Fig. (a).

We consider stationary contact problems for hyperelastic materials. The non-penetration conditions is discretized by a symmetric, pointwise sampling, essentially independent of the displacement discretization. This allows for a simple implementation, which is particularly convenient in the case of multi-body and self-contact where the a priori definition of master and slave sides is cumbersome. Nevertheless, usual error estimates can be shown. The drawback is, that the multiplier discretization is not of the special structure provided by dual mortar spaces. This prevents the use of efficient nonlinear Gauss-Seidel smoothers in two-body contact as it is exploited in monotone multigrid methods. Contact locking can be addressed by gap computation on a G^1-continuous boundary interpolation, see Fig. (b,c).

Instead, we use an augmented Lagrangian on the fine grid, and combine this with a primal multigrid hierarchy for the displacements. As a smoother, we employ overlapping nonlinear block Gauss-Seidel or Jacobi methods, and exploit the high arithmetic intensity of local QPs to be solved for effective parallelization. In order to have effective coarse grid corrections even in the case of sliding contact along rounded contact surfaces, we consider a level-dependent penalty factor [1].

The properties of the resulting contact solver are illustrated at several numerical examples.



(a) (b) (c)

Figure 1: (a) stress distribution in articulate cartilage (b) a concentric annulus test case (c) G^1 continuous boundary refinement

References

[2] F. Baumann, G. Duda, A. Schiela, M. Weiser. Identification of Stress in Heterogeneous Contact Models. In M. Hintermüller et al. (eds): *Non-Smooth and Complementarity-Based Distributed Parameter Systems*. Springer Nature, in press.

[1] R. Krause, M. Weiser. Multilevel augmented Lagrangian solvers for overconstrained contact formulations. In D. Auroux, J.-B. Caillau, R. Duvigneau, A. Habbal, O. Pantz, L. Pronzato, L. Rifford (eds): *ESAIM: ProcS*. 175–184, 2021.

Space under stress

Th. Ghesquière-Diericks[1], G. Anciaux[1], V. Acary[2], J.F. Molinari[1]

1: Department of Civil Engineering, EPFL, Lausanne, Switzerland
2: TRIPOP, INRIA, Center of University Grenoble Alpes, France

The increasing overcrowding of Earth's orbits has become a major concern. Commercial space launches are growing rapidly, and collisions or breakups of space objects lead to an uncontrolled proliferation of debris, further congesting orbital environments. This self-reinforcing process, known as the *Kessler syndrome*, poses a serious threat to space operations. To assess collision risks, space agencies such as NASA and ESA rely on breakup models, which in turn depend on empirical parameters. Physics-based models of dynamic fragmentation can significantly improve these inputs.

We have developed a high-performance finite-element framework with dynamic insertion of cohesive elements to capture crack initiation, propagation, branching, and coalescence. These dynamic fragmentation simulations provide detailed statistics of fragment masses, shapes, and velocities. We will discuss how this class of models compares favorably with analytical energy-based approaches for benchmark tests such as expanding rings or spherical membranes that mimic exploding fuel tanks. Fragment mass distributions depend strongly on geometry and loading conditions. In particular, high-velocity impacts produce power-law fragment size distributions (fig. 1a), and accurate velocity statistics require a robust treatment of fragments collisions and self-contact within partially damaged cohesive elements. In the spirit of CMIS, this presentation focuses on the associated numerical challenges.

A common approach for handling contact in explicit dynamics combines extrinsic cohesive elements with penalty-based contact. While effective over short time scales, we show that this method leads to exponential energy growth and artificial fragmentation at longer times. We present a systematic analysis of the sources of instability [1], identifying three dominant mechanisms: (i) excessively large initial cohesive stiffness, which restricts the stable time step; (ii) discontinuous stiffness jumps at the cohesive–contact interface; and (iii) discontinuities introduced by cohesive softening. Analytical error estimates, phase-space analysis, and energy growth metrics demonstrate that repeated switching between cohesive and contact states accumulates small per-step errors into a significant long-term energy drift. Within the explored parameter space, stability can only be maintained using time steps far below conventional limits. To reduce these artifacts, we evaluate an adaptive penalty strategy that links the contact stiffness to the evolving cohesive stiffness (fig. 1b). Although this approach restores energy conservation by removing stiffness discontinuities, it permits larger interpenetrations and is therefore better suited as a diagnostic tool than as a definitive solution. Overall, our findings show that penalty-based contact is not suitable for long-term, energy-consistent fragmentation simulations with physically meaningful fragment statistics.

To overcome these limitations, we propose an impulse-based contact formulation that explicitly accounts for the non-smooth nature of contact mechanics. The method is embedded implicitly within an explicit time-integration framework using the non-smooth Newmark-β scheme of Chen et al. [2]. Applied to one-dimensional dynamic fragmentation problems, this approach restores energy conservation and substantially improves robustness and accuracy in long-term simulations. Although the implicit contact resolution introduces additional computational cost, the enhanced stability enables larger time steps, resulting in overall performance comparable to—or even exceeding—that of penalty-based methods. These results demonstrate the potential of impulse-based contact as a reliable and efficient alternative for long-term dynamic fragmentation simulations.

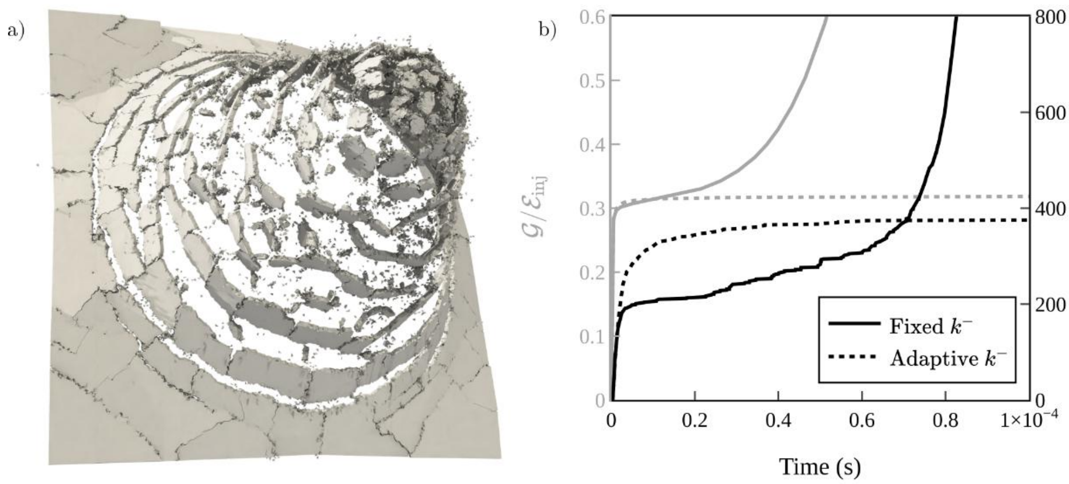


Figure 1: a) Finite-element simulations of debris creation due to impact using the open-source software Akantu. The image shows numerous cracks generated using extrinsic cohesive elements. The lead to small fragments at the impact zone, surrounded by concentric cracks. b) Comparison of the number of fragments (black) and the energy dissipation during decohesion process (grey), between the fixed contact penalty k^- approach (solid lines) and the adaptive one (dashed lines), that ties the penalty to the cohesive stiffness, thereby removing the cohesive-contact discontinuity. The adaptive approach removes the main source of instability, resulting in a stable plateau for both the number of fragments and energy dissipated, but comes with higher interpenetration, reducing the physical accuracy of contact.

References

[1] Th. Ghesquière-Diérickx, J.F. Molinari, G. Anciaux, Stability of extrinsic cohesive-zone model with penalty-based contact in explicit dynamic fragmentation simulations, *arXiv:2511.14323* , 2025.

[2] Q.Z. Chen, V. Acary, G. Virlez, O. Brüls, A nonsmooth generalized-α scheme for flexible multi-body systems with unilateral constraints, *International Journal for Numerical Methods in Engineering*, 96(8), 487-511, 2013.

Reduced-order compliant wheel–obstacle Contact interaction for rover mobility with experimental validation

Mahdi Maleki[1], Nicholas Kowalski[1] and József Kövecses[1]

[1]*Department of Mechanical Engineering and Centre for Intelligent Machines, McGill University, 817 Sherbrooke St. West, Montréal, QC, Canada*

Compliant wheels are preferred over rigid wheels in situations that require improved ground contact, such as off-road vehicles on rugged terrain. Simulating compliant wheel interactions is generally computationally expensive, often requiring the use of finite-element modeling to capture large deformations. This motivates the development of new methods to reduce the computational workload required to accurately model compliant wheels.

In this work, we present an efficient way of capturing the effect of deformation within a rigid wheel modelling framework by adjusting the contact parameters of a rigid wheel model. Instead of explicitly modelling for any structural deformation, the approach uses the constrained kinetic energy at the moment of contact to represent the effect of deformation in a simple and computationally efficient manner.

The method is based on projecting the motion of the rigid wheel into the space of constrained motion, following the formulation introduced in Ref. [1, 2]. With this projection, we can calculate the portion of kinetic energy aligned with the active constraint direction. The constrained kinetic energy is then used to calculate an equivalent radial deformation that happens during the contact with the obstacle using an effective stiffness. This deformation is used to update the contact point and the normal direction within the rigid wheel contact model, as illustrated in Fig. 1, allowing the rigid model to reflect the behaviour of a compliant wheel without modelling the tire structure.

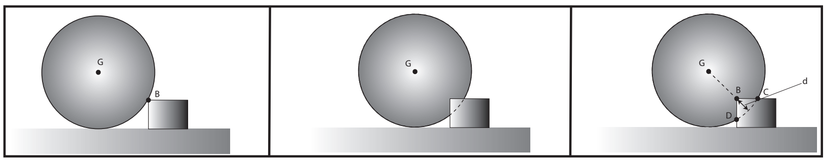


Figure 1: Schematic of deformation-based contact adjustment. The constrained kinetic energy provides deformation d, which is used to update the effective contact point from the obstacle edge to point C.

Building on the single-wheel formulation, we implement the reduced model in a complete rover multibody dynamics simulation. Each wheel of the rover uses the deformation-based update of the contact parameters, allowing the vehicle to reproduce compliant-wheel obstacle negotiation behaviour while retaining real-time performance. The model will be used to run obstacle traversal simulations across various obstacle heights, velocities, and stiffness values. In parallel, an instrumented rover platform is being used for experiments in which identical obstacle configurations are used to validate the simulation results by comparing wheel loads, motion, and climb outcomes.

References

[1] J. Kövecses, Dynamics of mechanical systems with motion constraints, *Journal of Applied Mechanics*, 75, 2008.

[2] M. Hirschkorn and J. Kövecses, Operational space formulation and kinetic energy projections in constrained multibody dynamics, *Multibody System Dynamics*, 32, 2014.

Surface Pattern Design using the Boundary Element Method and Bayesian Optimisation

Paulo Ricardo Ferreira Rocha[1,2], António Manuel Couto Carneiro[2], Francisco Manuel Andrade Pires[1,2]

1 - Department of Mechanical Engineering, Faculty of Engineering, University of Porto
2 - Institute of Mechanical Engineering and Industrial Management

Contact interactions, ubiquitous in engineering systems, drive energy dissipation through friction and adhesion. One of the main methods of controlling these phenomena is through surface topography. Nevertheless, manufacturing surfaces with highly controlled surface roughness and performing experimental testing are both difficult and costly. Hence, computational design and testing is a very cost-effective alternative, which allows focusing real-world production and validation efforts on the most promising designs.

Recently, patterned surfaces have emerged as a transformative approach to tunable contact, not only due to growing application demand, validated proofs-of-concept, and ease of manufacturing. The control of friction through surface design is also being actively investigated, targeting friction coefficient tuning, bio-inspired designs, or fully imposing a friction law [1]. These studies open new avenues for the development of systematic design frameworks in this emerging research field. In this context, the present work introduces a computational framework for automatic design of patterned surfaces, integrating the open-source boundary element code Tamaas with the in-house optimization tool, Piglot [2, 3].

In this initial stage, the focus is on designing deterministic surface patterns parametrized by a small number of variables. The objective is to enforce a prescribed contact area evolution for a given applied pressure. One of the investigated patterns consists of a grid of spherical asperities, similar to that used in [1]. The proposed strategy is evaluated using a range of objective functions, from single-value targets to full functional prescriptions. The results obtained in this first study lay the groundwork for more advanced applications, involving adhesive and frictional contact, as well as stochastic surface patterns.

References

[1] A. Aymard, E. Delplanque, D. Dalmas, and J. Scheibert, Designing metainterfaces with specified friction laws, *Science*, 383:200–204, 2024. doi:10.1126/science.adk4234.

[2] L. Frérot, G. Anciaux, V. Rey, S. Pham-Ba, and J.-F. Molinari, Tamaas: a library for elastic-plastic contact of periodic rough surfaces, *Journal of Open Source Software*, 5(51):2121, Jul. 2020. doi:10.21105/joss.02121.

[3] R. P. Cardoso Coelho, A. F. Carvalho Alves, T. M. Nogueira Pires, and F. M. Andrade Pires, Piglot: An open-source package for derivative-free optimisation of numerical responses, *Journal of Open Source Software*, 9:6652, Jul. 2024. doi:10.21105/joss.06652.

A mathematical framework for addressing boundary conditions in remodeling, hydrated soft tissues

Alessandro Giammarini[1], Andrea Pastore[2], Ariel Ramírez-Torres[3] and Alfio Grillo[2]

[1] *Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano - Piazza Leonardo da Vinci 32, Milano (MI), 20133, Italy.*
[2] *Dip. di Scienze Matematiche "G. L. Lagrange", Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129, Torino, Italia.*
[3] *School of Mathematics & Statistics, Mathematics & Statistics Building, University Place, Glasgow, G12 8QQ, UK.*

The mechanical behavior of living matter can be expressed by investigating the interactions between its constituents and the surrounding biological environment, resulting in systems in which the mechanical response is coupled, for example, to some processes that alter the inner structure of the medium, such as growth and remodeling. In principle, whenever two soft tissues are in contact, the anelastic processes in the tissues could be influenced by the interactions that are being exchanged through the boundaries.

In our work [1], we study the soft tissue as a hydrated, fluid-saturated porous medium undergoing an isochoric structural reorganization (remodeling) within a mathematical framework based on the Principle of Virtual Power in the presence of constraints. We consider the spatial gradient of the remodeling variable among the primary descriptors of our model, with the aim of formulating a theory that can provide insights on the tissue's behavior near the boundaries. In particular, we adopt Gurtin and Anand's theory of strain gradient plasticity [2] to describe remodeling, and we introduce a Darcy-Brinkman model for the fluid phase. In this way, we can provide suitable boundary conditions for the fluid and the remodeling variable that can be adjusted for describing contact interactions.

Finally, we emphasize some favorable computational aspects of the model, and we draw some parallelism with previous results of a model described by Darcy's law and a plastic flow without the strain gradient contributions in the remodeling descriptor [1, 3].

References

[1] A. Giammarini, A. Pastore, A. Ramírez-Torres, A. Grillo. A first-gradient approach to the remodeling and fluid flow in saturated porous media, *Mathematics and Mechanics of Solids*, 30:2185–2223, 2025.

[2] M.E. Gurtin, L. Anand A theory of strain-gradient plasticity for isotropic, plastically irrotational materials. Part II: Finite deformations, *International Journal of Plasticity*, 21:2297–2318, 2005.

[3] S. Di Stefano, A. Giammarini, C. Giverso, A. Grillo. An elasto-plastic biphasic model of the compression of multicellular aggregates: the influence of fluid on stress and deformation, *Zeitschrift für angewandte Mathematik und Physik*, 73, 2022.

Solving Frictional Contact Problems in Space and Time at Scale

R. Krause

King Abdullah University Of Science And Technology (KAUST) - Thuwal, Saudi Arabia.

We present an non-linear parallel solution method for frictional contact problems, which uses ideas from multiscale as well as domain decomposition methods. Our method is inherently non-smooth and allows for the massively parallel solution of frictional contact problems at scale.

Taking the locality of the (Coulomb) friction law and of the non-penetration con- straints into account, we first "separate" the non-smooth interface processes from smooth reaction of the material bulk by means of a multi-scale decomposition. Within this multi-scale decomposition, the non-smooth frictional effects at the interface are located on the finest scale. They can be dealt with locally by means of non-smooth solution approaches, thereby allowing for a highly accurate identification of the stick/slip regime. The smooth response of the bulk, however, is associated to the coarser scales of the model. In contrast to the fine scale effects, here global interaction has to be taken into account during the solution process. This is done efficiently by means of a modified multi-level basis inspired by linear multigrid methods. For the scale separation we employ a constrained Newton-like linearisation, which allows to "remove" the sticky node and to restrict the linearisation to where the underlying energy functional is differentiable.

We then show how our multi-scale approach can be combined with domain decomposition ideas, leading to a massively parallel solution methods for frictional contact problems.

In our presentation, we will explain the design of our multi-scale decompositon in detail and discuss its convergence properties. We will show that our approach is flexible in the sense that it also allows for the solution of, e.g., thermomechanical problems. We will furthermore exploit our method in the framework of a fully implicit time discretisation scheme for frictional contact problems, which is based on our stabilised Newmark method. Numerical examples will illustrate the performance and scalability of our method. We will in particular compare to standard approaches, such as active set strategies, and show that our approach is significantly faster.

References

[1] R. Krause, A Nonsmooth Multiscale Method for Solving Frictional Two-Body Contact Problems in 2D and 3D with Multigrid Efficiency, *SIAM Journal on Scientific Computing.*, 1399-1423, 2009.

[2] R. Krause, M. Walloth, A time discretization scheme based on Rothe's method for dynamical contact problems with friction, *Computer Methods in Applied Mechanics and Engineering.*, 1-19,2209.

Immersed domain approach for fluid-structure-contact interaction problems

Patrick Zulian[a,1,2], Maria G. C. Nestola[1], Hardik Kothari[1], Mhamad M. Alloush[3], Luca Mangani[3], Ernesto Casartelli[3], and Rolf Krause[1,2]

a. patrick.zulian@usi.ch

1. Euler institute, Università della Svizzera italiana, Lugano, Switzerland.

2. Faculty of Mathematics and Computer Science, FernUNI, Brig, Switzerland.

3. Competence Center of Fluid Mechanics and Hydro Machines, Hochschule Luzern, Switzerland.

The study of fluid-structure interaction (FSI) has gained significant traction in recent decades, with applications spanning various disciplines, including geophysics and biomedicine. In FSI, computational techniques are defined by the choice of discrete domain representation, falling into two main categories: "boundary-fitted" or "non-boundary-fitted" meshes. Boundary-fitted methods explicitly represent the fluid-solid interface and deform the fluid mesh in tandem with the solid mesh, typically within an arbitrary Eulerian-Lagrangian (ALE) framework. While these methods offer high accuracy, substantial solid deformations can severely distort the fluid mesh, leading to numerical problems and the need for remeshing. Non-boundary-fitted methods, on the other hand, maintain separate and non-matching fluid and structure meshes. The structure is described within a Lagrangian framework, while the fluid is typically described in a fully Eulerian framework. This flexibility demands higher mesh resolution to maintain comparable accuracy, necessitating the consideration of parallel computing.

We present an immersed domain approach for the numerical solution of fluid-structure-contact-interaction (FSCI) problems [1]. Within the overlapping volume, the fluid and structure are coupled, while mortar-based techniques are employed to couple different structures in contact on their surfaces. Specifically, we utilize dual Lagrange multipliers, which, within the nonlinear solution procedure, enable discrete field transfer using standard matrix-vector multiplication or storage of the linearized system of equations in a single matrix, thus facilitating algebraic multigrid strategies. We illustrate our general algorithmic framework and our primary parallel computing tools and discuss two studies conducted using variations of our approach.

The first study simulates the complete dynamics of a bio-prosthetic heart valve. We model the interactions between blood and the valve, blood and the aortic wall, and leaflets during valve closure. This solution strategy is specifically designed to address the contact problem using non-smooth methods, with solid and structure sub-problems solved in a segregated and iterative manner.

The second study simulates a diaphragm pump and the contact interaction between an elastic valve displaced by the fluid and the valve seats. This approach is monolithic, and penalty methods are employed to impose contact conditions.

References

[1] Nestola, M.G., Zulian, P., Gaedke-Merzhäuser, L. and Krause, R. Fully coupled dynamic simulations of bioprosthetic aortic valves based on an embedded strategy for fluid–structure interaction with contact. *EP Europace, 23 (Supplement 1), pp.i96-i104, 2021.*

Università
della
Svizzera
italiana

**Faculty
of
Informatics**

**CMIS 2026
Contact Mechanics
International Symposium**

21-24 April 2026
Continental Park Hotel Lugano