Extending Nitsche's method for the unilateral contact of beams

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Non-linearities and non-smooth behaviors caused by impact phenomena give rise to complex numerical problems in the form of variational inequalities that require appropriate and effective numerical methods. Numerical methods for contact problems have been an active area of research for many years, but new methods continue to emerge. One of them is the Nitsche method. Initially introduced by Nitsche [1] to impose Dirichlet conditions in a 'weak' manner, it has since been applied to various interface problems. Applications to 3D contact mechanics have been presented and mathematical analyses have been published (see, for example, [2, 3]). Unlike all other methods, Nitsche's method is variationally consistent (and converges optimally) and does not introduce any additional degrees of freedom. Despite the advantages of this method, the realistic contact forces it delivers are partly due to the use of a 3D stress tensor. The question of extending Nitsche's method to thin 1D and 2D structures therefore arises.

In this work, we extend Nitsche's method to beam contact, with an approach similar to what was done by Fabre et. al. [4] for plate models. In particular, we show that the classic Euler Bernoulli and Timoshenko beam models have kinematics that are too 'poor' in the thickness of the structure to benefit from the theoretical advantages of Nitsche's method. Indeed, for these two models, the Nitsche contact formulation obtained is equivalent to that of a penalty contact. We therefore propose a beam model with enhanced kinematics, introducing a term that models pinching in the thickness of the beam, and show that this allows us to implement a relevant Nitsche contact formulation. The relevance of the method is assessed by comparing its results with the classical Timoshenko model, where contact is implemented using the penalty method.

[1] J. Nitsche. Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind. *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, 36(1):9–15, July 1971.

[2] Franz Chouly and Patrick Hild. A Nitsche-Based Method for Unilateral Contact Problems: Numerical Analysis. *SIAM Journal on Numerical Analysis*, 51(2):1295–1307, January 2013.

[3] Franz Chouly, Mathieu Fabre, Patrick Hild, Rabii Mlika, Jérôme Pousin, and Yves Renard. An Overview of Recent Results on Nitsche's Method for Contact Problems. In Stéphane P. A. Bordas, Erik Burman, Mats G. Larson, and Maxim A. Olshanskii, editors, *Geometrically Unfitted Finite Element Methods and Applications*, volume 121, pages 93–141. Springer International Publishing, Cham, 2017. Series Title: Lecture Notes in Computational Science and Engineering.

[4] Mathieu Fabre, Cédric Pozzolini, and Yves Renard. Nitsche-based models for the unilateral contact of plates. *ESAIM: Mathematical Modelling and Numerical Analysis*, 55:S941–S967, 2021.