

A mathematical framework for addressing boundary conditions in remodeling, hydrated soft tissues

Alessandro Giammarini¹, Andrea Pastore², Ariel Ramírez-Torres³
and Alfio Grillo²

¹*Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano - Piazza Leonardo da Vinci 32, Milano (MI), 20133, Italy.*

²*Dip. di Scienze Matematiche “G. L. Lagrange”, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129, Torino, Italia.*

³*School of Mathematics & Statistics, Mathematics & Statistics Building, University Place, Glasgow, G12 8QQ, UK.*

The mechanical behavior of living matter can be expressed by investigating the interactions between its constituents and the surrounding biological environment, resulting in systems in which the mechanical response is coupled, for example, to some processes that alter the inner structure of the medium, such as growth and remodeling. In principle, whenever two soft tissues are in contact, the anelastic processes in the tissues could be influenced by the interactions that are being exchanged through the boundaries.

In our work [1], we study the soft tissue as a hydrated, fluid-saturated porous medium undergoing an isochoric structural reorganization (remodeling) within a mathematical framework based on the Principle of Virtual Power in the presence of constraints. We consider the spatial gradient of the remodeling variable among the primary descriptors of our model, with the aim of formulating a theory that can provide insights on the tissue’s behavior near the boundaries. In particular, we adopt Gurtin and Anand’s theory of strain gradient plasticity [2] to describe remodeling, and we introduce a Darcy-Brinkman model for the fluid phase. In this way, we can provide suitable boundary conditions for the fluid and the remodeling variable that can be adjusted for describing contact interactions.

Finally, we emphasize some favorable computational aspects of the model, and we draw some parallelism with previous results of a model described by Darcy’s law and a plastic flow without the strain gradient contributions in the remodeling descriptor [1, 3].

References

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