Interior-Point and Asymptotic Numerical Methods for Frictional Contact Problems

Vincent Acary, and Paul Armand

INRIA. TRIPOP team. Centre de l'Université Grenoble Alpes, France

Unilateral contact problems with Coulomb friction remain a central challenge in computational contact mechanics due to their intrinsic nonsmoothness, nonconvexity, and frequent rank deficiency arising from redundant constraints. Such difficulties are particularly acute in rigid multibody systems, granular media, and large assemblies of contacting bodies, FEM with structural elements (beams, plates and shells), where classical second-order methods often fail to converge robustly, while first-order methods—although reliable—suffer from slow convergence and limited accuracy.

In this work, we propose a *scalable second-order solution strategy* for frictional contact mechanics that combines a *primal–dual interior point method (IPM)*[1] with a path-following strategy based on the *Asymptotic Numerical Method (ANM)*[4]. The resulting approach, referred to as **IPM-ANM**, is specifically designed to address the challenges of nonsmoothness, hyperstaticity, and non-monotone solution paths that arise in discrete Coulomb friction models.

SOCCP formulation. Following standard time and space discretization of mechanical systems with unilateral contact and Coulomb friction, the governing equations are formulated as a Second-Order Cone Complementarity Problem (SOCCP)[3]. The contact reactions and relative velocities at each contact point are constrained by second-order (Lorentz) cones representing the Coulomb friction law. The resulting system is nonlinear, nonsmooth, and generally rank-deficient, as the contact configuration matrix is often not of full row rank. This framework allows the problem to be cast into a form amenable to interior-point techniques, while preserving the mechanical interpretation of sticking, sliding, and take-off contact states.

Interior-point framework and central path. The proposed solution strategy is based on a primal–dual interior-point method inspired by Mehrotra's predictor–corrector algorithm. A key theoretical contribution of this work is the proof of the existence of an analytic central path associated with the perturbed complementarity system. Under a Slater-type condition, we show that for any positive barrier parameter, the perturbed system admits a solution and that accumulation points of this path correspond to solutions of the original frictional contact problem.

This analysis provides not only a solid mathematical foundation for the interior-point algorithm, but also an alternative proof of solution existence for the SOCCP formulation of Coulomb friction. The interior-point iterations rely on semi-smooth Newton steps to handle the nonsmooth terms associated with friction, while maintaining iterates strictly within the interior of the friction cones.

Limitations of classical path-following. Despite its strong theoretical properties and good practical performance, the classical interior-point predictor–corrector strategy exhibits failures on certain problems, including relatively small systems. These failures are traced to a fundamental issue: the central path is not necessarily monotone with respect to the barrier parameter. As a result, enforcing a strictly decreasing barrier parameter—an implicit assumption of standard interior-point algorithms—may prevent convergence, even when a smooth solution path exists. This phenomenon is well known in numerical continuation and homotopy methods, but is rarely addressed explicitly in the context of frictional contact mechanics.

Key contribution: IPM-ANM coupling. To overcome this limitation, the main methodological contribution of this work is the integration of the Asymptotic Numerical Method (ANM) into the interior-point framework. Rather than advancing point-by-point along the central path, ANM constructs high-order local series expansions of the solution with respect to a continuation parameter. This allows the algorithm to capture entire branches of the central path, including regions where the parametrization with respect to the barrier parameter is non-monotone. The IPM-ANM approach reformulates the perturbed interior-point equations as a system with linear, bilinear, and constant terms, which is particularly well suited to ANM. At each continuation step, a sequence of linear systems—sharing the same Jacobian structure as the interior-point method—is solved to compute the coefficients of the asymptotic expansion. An estimate of the radius of convergence then determines the admissible continuation step. *Crucially, ANM removes the need for a strictly decreasing barrier parameter*, thereby restoring robustness in cases where classical Mehrotra-type algorithms fail. This represents a significant conceptual and practical advance for second-order solvers in frictional contact mechanics.

Numerical validation. The proposed methods are validated on a large benchmark set from the Frictional Contact Library (FCLIB)[2], including problems with up to several thousand contacts and highly redundant constraint configurations. The performance of the standard IPM and the enhanced IPM-ANM is compared against two widely used first-order methods: the Non-Smooth Gauss–Seidel (NSGS) method and the Alternating Direction Method of Multipliers (ADMM). The results demonstrate that:

- The interior-point method significantly outperforms first-order solvers in terms of accuracy and convergence rate on medium to large problems.
- Failures observed with the standard IPM are effectively mitigated by the ANM-based path-following strategy.
- IPM-ANM exhibits superior robustness on difficult, hyperstatic systems where both classical second-order and first-order methods struggle.

Conclusion. This work establishes *IPM-ANM as a robust and scalable second-order solver for frictional contact problems*. By explicitly addressing the non-monotone nature of the central path through asymptotic numerical continuation, the proposed approach bridges the gap between theoretical guarantees and practical robustness. The methodology opens new perspectives for the reliable simulation of complex contact-dominated mechanical systems, particularly in regimes where redundancy and nonsmoothness have traditionally limited the applicability of second-order methods.

References

[1] Vincent Acary, Paul Armand, Hoang Minh Nguyen, and Maksym Shpakovych. Second-order cone programming for frictional contact mechanics using interior point algorithm. *Optimization Methods and Software*, 39(3):634–663, 2024.

[2] Vincent Acary, Maurice Brémond, Tomasz Koziara, and Franck Pérignon. FCLIB: a collection of discrete 3D Frictional Contact problems. Technical Report RT-0444, INRIA, February 2014.

[3] Vincent Acary, Florent Cadoux, Claude Lemaréchal, and Jérôme Malick. A formulation of the linear discrete Coulomb friction problem via convex optimization. *ZAMM Z. Angew. Math. Mech.*, 91(2):155–175, 2011.

[4] Bruno Cochelin, Noureddine Damil, and Michel Potier-Ferry. The asymptotic-numerical method: an efficient perturbation technique for nonlinear structural mechanics. *Revue européenne des éléments finis*, 3(2):281–297, 1994.