Application of the Nitsche method to three industrial 3D elastoplastic frictional contact problems

C. Pozzolini(1), Y. Renard(2), M. Pignol(1), B. Souloumiac(1)

(1)Framatome, Zone Tech, 2 Rue Professeur Jean Bernard, 69007 Lyon. France
cedric.pozzolini@framatome.com,moise.pignol@framatome.com,bruno.souloumiac@framatome.com,

(2)INSA LYON, ICJ UMR5208, LaMCoS UMR5259, F-69621, Villeurbanne, France

Yves.Renard@insa-lyon.fr

In recent decades, Nitsche's method has emerged as a powerful and elegant alternative for the numerical treatment of contact between deformable solids. Originally introduced in 1971 by Nitsche [1] for the weak imposition of boundary conditions, it has since been extensively applied to interface-coupled problems. Its first application to contact mechanics was presented in 2008 by Wriggers et al. [2], but it was an non-consistent formulation. Another complete mathematical analysis of linearized elasticity provided las early as 2013 by Chouly, Hild, Renard et al. unlocked the problem with a new consitent formulation [3,4,5]. The principal advantage of Nitsche's method is that it is variationally consistent, ensuring optimal convergence rates, without introducing additional degrees of freedom like the Lagrange multiplier method. This eliminates the need to satisfy a discrete inf-sup condition. This advantage comes at the cost of having to evaluate the boundary traction from the continuum stresses. To enhance stability, advanced symmetric and skew-symmetric variants have been developed, the latter being stable for any positive penalty parameter. The method has been successfully extended to more complex scenarios, including finite deformation elasto-plastic contact. In this context, the approximation strategy proposed here, which implements a weak integral contact condition conceptually similar to Lagrange multipliers but within the Nitsche framework, was first developed and validated in the open-source finite element library GetFEM [6,7]. It has been applied to a wide range of contact scenarios, including small and large deformations for elastic and hyperelastic materials, with and without friction.The goal of this note is to describe the application of Nitsche's method to enforce contact conditions, with or without Coulomb friction, between two elasto-plastic bodies. after that, we present how this method has been operationalized within the industrial finite element software SYSTUS/SYSWELD 2024 to solve practical engineering problems, demonstrating its effectiveness within the small deformations' framework. In this work, we employ segment-to-segment integration on each slave element and enforce contact constraints at each slave element. and the contact detection is done at Gaussian points, which is very advantageous.

References

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