

Analytical Modeling and Experimental Validation of Ball–Beam Impact Dynamics Using a Nonsmooth Contact Dynamics Framework

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Impacts between rigid bodies and flexible structures arise in many engineering applications and can exhibit complex contact phenomena that challenge current modeling approaches. This work considers a benchmark configuration of a rigid ball falling on a simply supported Euler–Bernoulli beam. The objective is to formulate and validate an integrated set of analytical, experimental, and numerical results for this benchmark configuration, with particular emphasis on identifying (i) when and why sub-impacts occur and (ii) under which conditions the simplified model (rigid ball – elastic beam) is applicable.

An analytical model is developed using a piecewise approach that separates the motion into distinct phases. During the free-flight phase, the ball moves independently while the beam response is governed by modal analysis. The contact phase is characterized by coupled dynamics in which the ball is modeled as a point mass at the contact location, enabling unified ball-beam motion [1]. Contact constraints govern transitions between phases: impact begins when the bodies touch, while separation occurs when contact is lost.

Experimental validation is achieved through a carefully designed setup that ensures repeatable initial conditions. A steel ball is released from controlled heights using an electromagnet (Figure 1a). Structural response is captured using a stereo Digital Image Correlation system (ZEISS ARAMIS 4M), enabling detailed kinematic reconstruction and precise measurement of displacement, velocity, and acceleration histories.

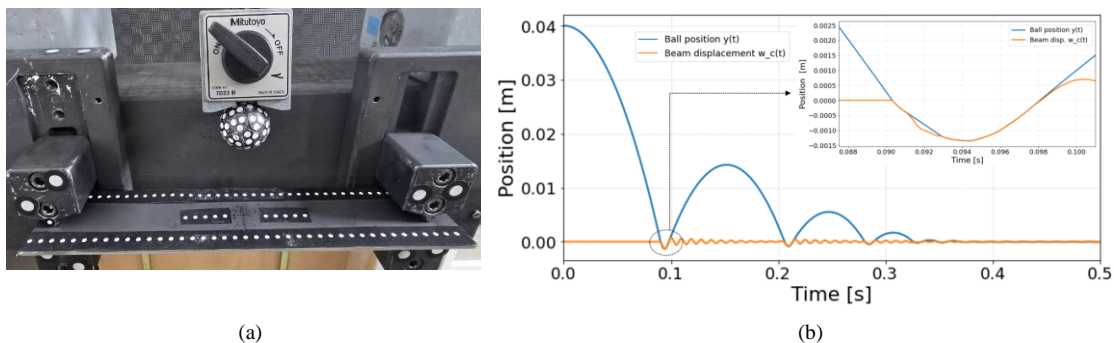


Figure 1: Ball and beam model: (a) experimental setup, (b) ball and midspan beam displacement over time, highlighting sub-impacts.

In parallel, numerical solutions of the governing equations are obtained using a finite-element beam model combined with a nonsmooth contact dynamics (NSCD) Moreau–Jean time-stepping scheme (Figure 1b) [2]. The numerical approach can be extended to a wide range of significantly more complex configurations, which motivates its investigation in this work. Through systematic comparison of all three methodological approaches on this benchmark problem, the work clarifies the capabilities and limitations of analytical models and current NSCD numerical schemes for rigid–flexible impact dynamics.

References

- [1] Goldsmith, W. (1960). *Impact: The Theory and Physical Behaviour of Colliding Solids*. Edward Arnold Publishers Ltd., London.
- [2] Jean, M. (1999). “The Non-Smooth Contact Dynamics Method.” *Computer Methods in Applied Mechanics and Engineering*, 177(3–4), 235–257.