Influence of numerical schemes on the apparent dynamics of the contact between a viscoelastic beam and two rigid stops

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In numerous applications of nuclear engineering and structural mechanics, many slender structures—notably steam generator tubes, fuel rods, and control rod drive mechanisms—can come into contact with rigid walls or abutments under dynamic loading. Accurate modeling and analysis of these phenomena are essential for evaluating the reliability, lifespan, and vibration resistance of these components under normal or accidental (earthquake) operating conditions.

Vibrating systems subjected to unilateral stresses exhibit inherently non-smooth dynamics characterized by the occurrence of impacts. The existence of chaotic regimes has been clearly established in several oscillator configurations subjected to abutments. However, for continuous beams, these phenomena are much less frequently described in the literature. Among the relevant contributions are the work of Liakou, Denoël, and Detournay [1] on the fast dynamics of beams with unilateral constraints, and the mathematical analysis by Mbengue [2], which provides existence and uniqueness results for a nonlinear beam model in contact with a foundation.

In industrial applications, contact is often modeled using a penalty method, allowing for low interpenetration and simplifying numerical processing. However, it is also possible to strictly impose non-interpenetration, leading to a contact force implicitly defined by the complementarity conditions. For the case of point obstacles, the work of Kuttler and Shillor [3] provided a detailed analysis of regularity and solutions. The present work is placed within a framework extended to the case of continuous and regular obstacles for a viscoelastic Euler-Bernoulli beam.

Semi-discretization using finite elements leads to differential inclusion, for which we exploit the singular mass method, initially developed by Renard [6] and adapted to thin structures by Pozzolini, Renard, and Salaün [7]. The beam is discretized in space using Hermite finite elements, necessary to handle a fourth-order spatial derivative operator. We then analyze the influence of the time scheme on the dynamics, in particular the effects related to scheme parameters, the penalty, and mass matrices. From a numerical point of view, several approaches coexist to enforce non-interpenetration: the penalty method, Lagrangian methods (pure or augmented) with complementarity conditions, or hybrid methods. Recall that convergence towards the reference solution is observed when the penalty coefficient remains within a range that guarantees both mechanical consistency and numerical stability. Beyond a certain threshold, the increased stiffness of the contact induces numerical oscillations and a loss of dynamic fidelity. This is one of the reasons that led us to investigate the emergence of non-repeatability during vibro-impact calculations. Therefore, we analyze here a simplified model of a beam subjected to repeated impacts, induced by an imposed sinusoidal displacement. Newmark schemes, whose stability in non-smooth contexts is still poorly documented in the literature, serve as the basis for the time integration [5].

We show that the observed dynamics—in particular the appearance or disappearance of bifurcations—depends crucially on the numerical choices. The objective of this study is therefore to provide a robust framework for the analysis of the non-smooth dynamics of beams in rigid contact, and to establish criteria for correctly interpreting the simulations.

References

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