A Point-Wise Complementarity Model for Frictional Contact Problems

Qingfeng Lou and József Kövecses

Department of Mechanical Engineering and Centre for Intelligent Machines, McGill University, Montréal, Canada

Contact problems in dynamic simulation remain challenging due to three intrinsic characteristics of the Coulomb friction law: coupling between normal and tangential forces; complementarity between velocities and forces; and the nonlinearity arising from friction forces that may point in any tangential direction while remaining bounded in magnitude.

The second order cone complementarity problem (SOCCP) model provides a mathematically consistent representation of ideal friction cone. However, it introduces a nonlinear augmented velocity term to construct the dual cone pairing with forces. Moreover, the three physical motion phases of a contact point: free (open), stick, and slide, are represented collectively within a single vector-wise orthogonality condition rather than being distinguished by separate inequality relations. These model level limitations propagate to associated fixed point and Newton type solvers, making convergence and stability difficult to ensure in practical computation.

The linear complementarity problem (LCP) model applies certain simplifications relative to SOCCP. A contact point is decomposed into one normal and two orthogonal tangential directions, and the friction bounds along these directions are predefined as fixed values. Thus each scalar component is treated independently and constrained to be nonnegative, reducing the vector-wise orthogonality to element-wise complementarity. As a result, the model corresponds to a friction cuboid, which neglects the intrinsic coupling and nonlinearity of Coulomb friction. These model level limitations propagate to associated LCP solvers such as Lemke, Murty, Judice, and classical projected Gauss–Seidel.

We introduce a model that provides a more physically coherent representation of frictional contact. It contains no augmented velocity term as in SOCCP and instead employs the constraint force and velocity pair directly. The model retains the decomposition into one normal and two tangential directions as in LCP, but the tangential bounds are defined as functions of the normal force and scaled by the tangential force or velocity directions. The force and velocity pair does not form a dual cone with vector-wise orthogonality and also breaks the element-wise complementarity structure. The model therefore lies outside both SOCCP and LCP, representing a contact point-wise complementarity with a direct physical interpretation. Its behavior is equivalent to an adaptive friction pyramid that becomes identical to the ideal friction cone at the computed solution, capturing the coupling, complementarity, and nonlinearity inherent in the Coulomb friction law.

The model can be solved through a two stage procedure: first solving the contact problem with a classical LCP solver such as projected Gauss–Seidel, then upon convergence classifying the motion phases and applying the rescaling needed to obtain the adaptive pyramid consistent with the ideal cone. The approach is self-contained, requiring no predefined friction bounds or external corrections from the dynamics model. It preserves the stability of standard LCP solvers while eliminating the coupling and nonlinear errors. We further implemented the approach within Vortex Studio, integrating it with its matrix data structures and achieving improvements in both physical accuracy and computational performance.