

Shifted–Penalty Multigrid Method for Contact

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High-performance computing is essential for efficiently solving large-scale contact problems. Simulating such phenomena at engineering scale is often limited by computational resources, making it crucial to design algorithms that fully exploit modern hardware like multi-core CPUs and GPUs. Iterative solvers and preconditioners play a central role in this efficiency.

Monotone Multigrid (MMG) methods offer optimal complexity and robustness. In parallel, Penalty and Augmented Lagrangian methods handle over-constrained and fuzzy constraints effectively. Among these, the Shifted-Penalty method is notable for accurately enforcing constraints while remaining competitive with non-smooth techniques like the semi-smooth Newton method.

To combine the optimal complexity of MMG with the flexibility of shifted-penalty methods, we introduce the Shifted-Penalty Multigrid (SPMG) method. Designed from the ground up for GPU architectures, SPMG unifies nonlinear smoothing with constraint-aware multigrid strategies.

Our implementation uses matrix-free differential operators and memory-efficient semi-structured meshes to discretize elasticity equations. We present the SPMG algorithm with a focus on nonlinear smoothing and constraint coarsening.

We evaluate performance on the Grace-Hopper superchip of the CSCS Alps supercomputer. Emphasis is placed on single-node GPU performance, kernel design, and convergence behavior in simple contact scenarios. Finally, we demonstrate SPMG’s scalability on large-scale problems with hundreds of millions of degrees of freedom.

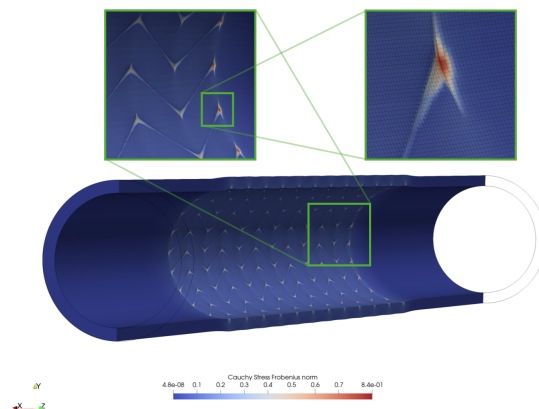


Figure 1: High resolution stress computation for nonlinear contact problem - 268 319 520 dofs on Apple M1 Max

References

- [1] G. Zavarise, The shifted penalty method, *Computational Mechanics.*, 56:1–17, 2015.
- [2] G. Marchi, H. Kothari, R. Krause, A. Nelson, P. Vassilevski and P. Zulian, Shifted–Penalty Multigrid Method for Contact, *In preparation*.