# Comparison of three algorithms on frictional contact problem with two distinct solutions

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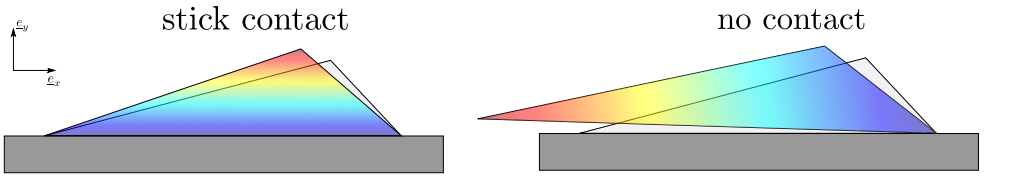
The Coulomb law of friction is widely used to model the contact between two solids. It is well-known that mathematical results on the existence and uniquement of solution for contact problems with Coulomb friction are limited to small friction coefficient [1]. This is due to the fact that this law cannot be derived from a potential but only from a bi-potential. As a consequence, some contact problems may have one, one or multiple (up to infinity) solutions.

In this paper, we consider the quasi-static frictional contact between one elastic body and one rigid obstacle. This two-dimensional academic example coming from [2] exhibits multiple solutions when the friction coefficient $\mu \geq 3$. Indeed, one solution with no contact and one solution with stick contact, illustrated in Figure 1, both verify the problem's equations.




$\underline{u}_{ex,1} = -6\alpha y \underline{e}_x - \frac{3}{4}\alpha y \underline{e}_y$ $\qquad$ $\underline{u}_{ex,2} = (7x + y - 7)\alpha \underline{e}_x + \alpha(1 - x - \frac{7}{4}y)\underline{e}_y$

Figure 1: Two solutions for problem [2]

Many numerical methods are available to solve contact problems involving friction [3]. Here, we choose 3 methods: the augmented Lagragian technique with Uzawa algorithm [4], the Nitsche method [5] and the primal-dual method [6] (convex optimization approach with cone constraints). We aim at studying the influence of the parameters of each method (usually defined by the user) on the nature of the converged solution. We show that both the initialization fields and the values of penalty parameters or step length do have an influence on the nature of the obtained solution and that the no contact solution is more likely to be found. It highlights the crucial need of verification tools and guidelines dedicated to users for such numerical methods.

## *References*

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