

Mixed-dimensional beam-to-solid interaction: From embedded fibers to contact

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Interactions between slender fiber- or rod-like components and three-dimensional solids are central to many mechanical systems in engineering and bio-mechanics. Numerical models typically rely on the finite element method and often require a trade-off between accuracy and computational cost. For fiber-reinforced materials, two common strategies exist: (i) homogenization, where fiber and matrix stiffnesses are combined into an anisotropic material law, reducing meshing effort but limiting insight into local fiber-matrix interactions; and (ii) fully resolved 3D modeling, which captures detailed behavior but is computationally expensive. The main idea of the presented framework is to provide finite element formulations to combine the successful and rich history of nonlinear beam theories based on 1D-Cosserat continua with classical 3D continuum finite elements. This allows to explicitly model the fibers while still maintaining a moderate overall model complexity compared to the fully resolved model. The resulting problems are referred to as beam-to-solid (BTS) interaction problems.

Because beam and solid equations differ in dimensionality, BTS problems are inherently mixed-dimensional. Coupling a 1D beam to a 3D solid introduces a line-load-type singularity, whose influence on the global response is negligible as long as the beam cross-section is smaller than the solid elements. A second assumption is that fiber volume is not removed from the matrix, producing overlapping domains. For typical fiber-matrix stiffness ratios, this inconsistency is negligible and greatly simplifies meshing, allowing independent discretizations and non-matching grids.

The BTS framework can be applied to various beam and solid formulations. Interaction constraints are enforced along the beam centerline using a weighted mortar approach with penalty regularization, eliminating Lagrange multipliers and yielding robust coupling. Special attention is required when coupling rotational degrees of freedom, which demands a suitable triad field within the solid. For beam-surface interactions, the surface normal also enters the constraint formulation. This can include problems where the beam is coupled to the solid surface and problems where the beam is in unilateral contact with the solid surface.

Among the main topics addressed in this talk are a detailed outline of the beam-to-solid interaction framework, as well as a discussion of the characteristic traits of mixed-dimensional interaction in solid mechanics. Selected quantitative and qualitative examples are presented to highlight in order to underline the usability for real life science and engineering applications

References

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