Multilevel Augmented-Lagrangian Methods for Overconstrained Contact Discretizations

M. Weiser[1], F. Baumann[1] and R. Krause[2]

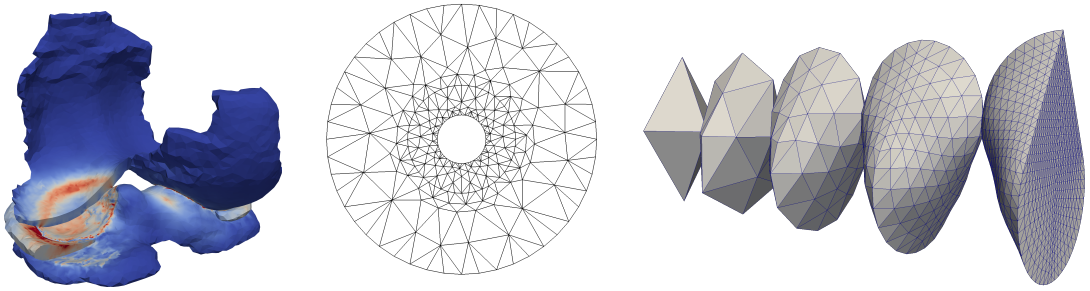
[1] {*weiser,felix.baumann*}*@zib.de, Zuse Institute Berlin, Takustr 7, 14195 Berlin, Germany*
[2] *rolf.krause@kaust.edu.sa, King Abdullah U. Sci. Tech., Thuwal 23955-6900, Saudi-Arabia*

The talk will address overconstrained formulations for multi-body contact and adapted multilevel solvers of augmented Lagrange type for the resulting QPs, with application to stress identification in articulate cartilage [2], see Fig. (a).

We consider stationary contact problems for hyperelastic materials. The non-penetration conditions is discretized by a symmetric, pointwise sampling, essentially independent of the displacement discretization. This allows for a simple implementation, which is particularly convenient in the case of multi-body and self-contact where the a priori definition of master and slave sides is cumbersome. Nevertheless, usual error estimates can be shown. The drawback is, that the multiplier discretization is not of the special structure provided by dual mortar spaces. This prevents the use of efficient nonlinear Gauss-Seidel smoothers in two-body contact as it is exploited in monotone multigrid methods. Contact locking can be addressed by gap computation on a G^1-continuous boundary interpolation, see Fig. (b,c).

Instead, we use an augmented Lagrangian on the fine grid, and combine this with a primal multigrid hierarchy for the displacements. As a smoother, we employ overlapping nonlinear block Gauss-Seidel or Jacobi methods, and exploit the high arithmetic intensity of local QPs to be solved for effective parallelization. In order to have effective coarse grid corrections even in the case of sliding contact along rounded contact surfaces, we consider a level-dependent penalty factor [1].

The properties of the resulting contact solver are illustrated at several numerical examples.



(a) (b) (c)

Figure 1: (a) stress distribution in articulate cartilage (b) a concentric annulus test case (c) G^1 continuous boundary refinement

References

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